WORKING
PAPERS

MERGERS AND FREE RIDERS
IN
SPATIAL MARKETS

James D. Reitzes
and
David T. Levy

WORKING PAPER NO. 178

May 1990

FI'C Burl'!ll of Erooomics working papers are preliminary materials circulated to stimulate discussion and critical comment All data contained in them are in

Mergers and Free Riders in Spatial Markets

James D. Reitzes*
Federal Trade Commission

and

David T. Levy
University of Baltimore

March, 1990

Abstract: Prior analyses have found little incentive for merger in the absence of efficiency gains. Ei ther merger is unprofitable, or outside firms earn higher profits than the merged parties. In both cases, firms have incentives to free ride on the merger activity of rivals. We examine merger in a model with differentiated consumers, and find that mergers are profitable. Moreover, the free-rider problem is largely eliminated under uniform pricing; it is completely eliminated under discriminatory pricing.

* Federal Trade Commission, Bureau of Economics, 6th & Pennsylvania, NW, Washington, DC 20580.

Mergers and Free Riders in Spatial Markets
1. Introduction
Past theoretical analyses find limited gains to merging purely for anticompetitive purposes. Salant, Switzer and Reynolds (1983) show that, in a quantity-setting game with homogeneous goods and constant marginal costs, a merger will generally lower the combined profits of the merged parties. On the other hand, Deneckere and Davidson (1985) discover that the formation of a coalition increases the profits of the combined parties in a price-setting game with differentiated products. 1 However, all outside firms earn higher profits than the merged parties. Firms therefore have an incentive to refrain from merger themselves, and instead free ride on merger activity by their rivals.2 This paper focuses on the free-rider problem by examining merger in a market with differentiated consumers (i.e., a spatial market). When firms participate in a spatial market, they have considerable incentive to merge even in the absence of efficiency gains (see Farrell and Shapiro (1990a,1990b) for mergers that generate efficiencies).3 The free-rider problem is largely eliminated under uniform

1 A coalition may differ from a merger. A coalition is formed for mutual gain, but can always disband into individual noncooperative players with separate profit-maximizing goals. A merged entity maximizes combined profits, unless broken into separate players through the sale of a portion of its assets. Thus, a merged firm cannot credibly act as separate players, except in spatial models when the merger chooses nonneighboring locations (see below).
2 Deneckere and Davidson (1984) find similar results when examLnLng collusive behavior in a model with homogeneous goods and capacity constraints. 3 Recent literature

pricing; and, it k completely eliminated in the case of discriminatory pricing.
The paper is organized as follows: section 2 describes the basic model, section 3 presents a two-firm merger with uniform pricing, section 4 considers a multifirm coalition with uniform pricing, section 5 examines discriminatory pricing, and section 6 offers concluding remarks including a discussion of relocation and entry. In the section on multifirm coalitions, we find that the coalition desires to use a "sequential limit pricing" strategy that bears similarity to a basing-point system. However, the tendency to use this strategy may preclude the possibility of a pure-strategy equilibrium in certain cases.

2. The Hodel
In our model, N firms and a continuum of consumers are located around
a circle. Distances are measured in units of �2�~� radians, implying that the circle is of unit circumference. The location of a given firm or consumer is identified by its equivalent arc measure in �2�~� radians. Firms are evenly-spaced, located at positions, ljN, 2/N, N/N. For reference purposes, firm i denotes the producer located at position i/N. Consumers are spread uniformly around the circle, and w denotes the buyer located at position wiN. By definition,

between any two locations, ilN and j/N. 4
Each consumer may purchase a bundle containing a variable amount of a homogeneous good and one unit of a differentiated good. If consumer w's bundle contains m units of the homogeneous good and a single unit of the differentiated good offered by firm i, then its utility is expressed as follows:

U(m,i,w) - m + (a - td(i,w», where t is unit transport cost.s
In the above function, m represents the subutility derived from the homogeneous product, and (a - td(i, w» represents the subutility derived from variety i. Without loss of generality, we assume that the price of the homogeneous good equals one, and the price of variety i equals pi. Letting 1 represent argmini pi + td(i, w), a utility-maximizing consumer would purchase one unit of variety 1 whenever a - (pi + td(i,w» > O. In the equilibria considered below, we assume that this inequality holds for all consumers. If all N firms produce positive output levels, then this

to be sunk to location (e.g., due to the establishment of immobile physical assets or the maintenance of reputation for a given brand location). We further assume that diseconomies preclude a

p* - Z; �~�*� - t/Nz - f. (4)
The equilibrium is sustainable under threat of entry if t/(N+l)z - f < O.

3. The Merger of Two Firms with Uniform Pricing
Consider a merger of two firms, i' and i'+l. The merger generates no efficiency gains; marginal and fixed costs are unaffected. However, the acquisition of firm-specific capital through merger implies that the merged parties can and will produce two brands. We further assume that substantial location-specific investment makes relocation a prohibitively costly strategy during the period under consideration. Entry is also precluded. The implications of these assumptions are discussed in the concluding section.
We first solve the profit-maximization problem for the merged firm, which can be expressed in the following manner:

max i' i'+l �~� IE �~�i�'� + �~�i�'�+�l� P ,p
(pi' _ c) [(pi'-l + pi'+l _ 2pi' + 2t/N)/2t]
+ (pi'+l _ c) [(pi' + pi'+Z _ 2pi'+1 + 2t/N)/2t] - 2f. (5)

The first-order conditions yield the following results:
pi' _ (1/4) (pi'-l + 2pi'+1 + Z + tIN)
pi'+l _ (1/4) (pi'+Z + 2pi' + Z + t/N) ,

(6)
(7)

distances, no pure-strategy

If pi'-l - pi'+2, then equations (6) and (7) are symmetric. The merged parties would charge the same price. All outside firms face a first-order condition expressed by equation (3), and that equation is symmetric with respect to the prices of neighboring firms. By recursive application of these first-order conditions, it can be shown that whenever pi' _ pi'+l, firms equidistant from the merger would charge the same price (see Appendix A). Hence, an equilibrium does exist where pairs of firms act symmetrically.
We now introduce notation to distinguish firms based on their location relative to the merged parties. Let pO represent the price charged by a merged firm, and pk represent the price charged by an outside firm that lies at distance kiN from the closest merged party.
merged parties use the following reaction function:

Under symmetry, the

po _ (1/2)(pl + Z + tiN). (8)
When all nonmerging firms satisfy their first-order conditions, firm k's reaction function can be expressed as follows (see Appendix A):
pk _ (BK-k/BK)pO + «BK _ BK-k)/BK)Z, (9)
where K ... (N-M)/2 «N-M+l)/2) if N-M is even(odd)
M ... number of merging firms,

Bn E 4Bn - l - Bn -2, BO - 1, B-1 - 1(2) if N-M is even(odd).

Letting pk _ pl in the above equation, we can now solve (8) and (9) simultaneously to obtain equilibrium values for pO and pl. By recursive

6

application, we can then obtain the equilibrium values for all prices:7
pO Z + (BK/L) (t/N) (10)
Z + (BK-k/L) (t/N) (11)
where L .. 2BK _ BK-l.
Given the initial conditions and the defined behavior of Bn, we can assert that BK > BK-l > ... > BO > O.
The profits of the merged parties can be described as follows: 8
?f0 I!!! �~�/�M� - [(po - c)2/2tJ - f

- [1 + (BK/L)J2(t/2N2) - f. (12)
For any outside firm, profits can be represented in the following manner:9
[ pk _ C ) 2/ t ] - f (l3a)
[1 + (BK-k/L)]2(t/N2) - f. (l3b)
It can be shown that ?fl > ?fo > ?f2.l0 Using the subsequent definition,

7 In spatial models with linear transportation costs, a firm must consider a market-appropriation strategy where it sets a sufficiently low price to capture its neighbor's entire market. (This can be accomplished by setting price tiN + € below that of the neighboring firm.) If a given firm finds that market appropriation dominates market sharing at the price vector representing the fixed-point solution under market-sharing strategies, then a pure-strategy Bertrand-Nash equilibrium will not typically exist. In the case of 2-firm mergers, firms do prefer to share

the market at the fixed-point solution described below be(a

we summarize our findings concerning post-merger prices and profits.11

Definition:
A given pair of firms are considered "neighbors" if there exists a nonempty set of consumers where these firms represent the two closest producers.

Proposition 1:
If two neighbors merge, then post-merger price behavior implies that
pO > p1 > ... > pK > p*. All outside firms charge lower prices than the merged firms, and prices decline as the distance from the merger increases. Further, �~�1� > �~�o� > �~�2� > ... >

Deneckere and Davidson (1985) appears limited. A firm would only avoid merger when close rivals are expected to merge within a given time period, and some constraint limits the number of viable mergers in the market.
By reexamining the profit-maximization problem shown in equation (5),
it is easily shown that firm i' only gains from merging with either of two firms: i '+1 or i'-1. If firm i' merges with any other firm, the first-order condition for either of the merged parties is still represented by equation (3). Hence, conditions are unchanged from the pre-merger equilibrium. In a Bertrand-Nash equilibrium with uniform pricing, mergers would only occur between neighboring firms in the absence of efficiency gains.
Of course, incentives may exist for merging with nonneighboring firms under more strategic interaction (see Braid (1986) and Levy and Reitzes (1989». For instance, consider a Stacke1berg 1eader-follower game. A merger among nonneighboring firms produces no benefits from coordination if the merged parties act as followers (i.e., price takers). Instead, let the merged parties act as leaders. If one merged party raises its price, then the followers choose a higher price level. As a consequence, the profits of the its

denote a coalition member located sjN from the closest border, when that border lies in a (counter)clockwise direction. That coalition member charges a price, PS'(ps·,). Further, DS'(DS ') _ pS'_ps-l' (Ps"_pS-l') when

° < s :5 S-1. Two other definitions are needed. First, DO' (Do") ... pO' _pl'
(pO" _pl"), where pO' (po") represents the price set by the border firm, and pl' (pl") represents the price set by the neighboring outside firm. Second,

the term, DSs ... (p1" + �~�_�o�S�-�l� on") - (p1' + �~�_�O�S�-�l� Dn'), represents the difference in prices set by the two most-insulated coalition members. The profit-maximization problem of a multifirm coalition can be expressed as follows: 15

maxDs, Os" (for s - 0,1, ... S-l) ,
1rM -2:: S-2 s - ° (pl' + �~�_�s�o� On' - c) [ (_DS' + DS+1' + 2t/N)/2t)
+ 2:: S-2 �S�~�O� (p1" + �~�_�s�o� Dn" - c) [ ( - DS" + Os+1" + 2t/N)/2t)
+ (p1' + �~� S -1 - ° Dn' - C) [( _OS-1' + DSS + 2t/N)/2t)
+ (p1" + �~�_� So -1 On" _ C) [( _DS-1" - DSS + 2t/N)/2t) - Mf
subject to

DS :5 t/N; _DSS:5 t/N. (14)

The following first-order conditions may be derived:

� d�~�/�d�O�O�'� - (l/t) [_DO' + S(t/N) - (1/2) (p1' -c) + DSS) + ).0'. (lSa)

15 The constraints are needed to preclude one coalition member from appropriating its neighbor's entire market. Thus, the price differential between two neighboring firms can never exceed t/N. By assumption, a consumer buys from the closer firm when two firms set mill prices that result in equivalent prices inclusive of transportation costs. This represents the limiting case of a situation where, in order to avoid market appropriation, a firm would never set its price more than (t/N - f) above that of a neighboring rival. Letting �f�~�O�+�,� we obtain the above result.
11

d,f!/dOS ' - (lit) [-OS' + (S-s) (tiN) + OSs] + ).s' for all s �~� 0 (15b) Comparable first-order conditions exist with respect to D°" and Os".
If outside neighbors behave symmetrically, then p1' _ p1" - pl. We
can now describe the solution to the above first-order conditions, based solely on the distance that a coalition member lies from its closest border:

D° - (tiN) (p1_C) �~� 2(S-1)(t/N) (M-2)(t/N)
- S(t/N) - (1/2)(p1-c); (p1_c) > 2(S-1)(tjN) (M-2)(tjN) (16a)
OS - tiN. (16b)

(tiN)

)j (

prices inclusive of transportation costs are identical to those of a "basing-point" system, where a base lies at each border. A basing-point system may thus indicate the operation of an effective coalition.
To show nonexistence of a pure-strategy Bertrand-Nash equilibrium,
note that the coalition engages in sequential limit pricing if pl S c + + the

in the following manner. If w 1 M, then p - c. If w E M, then pew) - c +

t(min [(d*(i' ,W),d*(j' ,w)] }. The coalition

discriminatory pricing is the preferred practice when firms choose their pricing policy in a noncooperative fashion.
Since firms offer identical products and act as Bertrand competitors,
any

o

pi(W) - C + td(j*(i,w),w)
- c + td(i,w)

if d(i,w) < d(j*(i,w),w)
if d(i,w) �~� d(j*(i,w),w)

(18a)
(18b)

In the future, we let j*(i l,i2, ••. ,in,w) represent argminj d(j,w) for
j �~� i l,i2, ••. ,in.
Consider a merger between two firms, i' and i'+n, where I'. (i' ,i'+n). When either merged firm has a cost advantage over all outside firms, the best strategy of the merger is to merely outcompete the low-cost outside

rival. Let i*(I' ,w) • argmini _i, ,i'+n d(i,w). Of the two merged parties,

i*(I' ,w) denotes the low-cost producer for serving customer w. If outside firms follow the strategy described in equation (18), the following price schedule represents the best response for the merged parties:
pi*(W) - C + td(j*(I' ,w) ,w) if d(i*(I' ,w) ,w) < d(j*(I' ,w) ,w) (19a)
- c + td(i*(I' ,w) ,w) if d(i*(I' ,w) ,w) �~� d(j*(I' ,w) ,w). (19b)

When firms use the above strategies, any consumers that are closest to
the two merged parties would face a higher post-merger price. Consider a set of consumers, Wl - (w: d(i' ,w) S d(i'+n,w) < d(j*(I' ,w),w)}.
For wl E Wl, firm i'

- c + td(j*(i' ,i'+n,wl),wl) > c + td(i'+n,wl). (21)
In serving consumers that belong to Wl, the revenue of firm i' increases subsequent to the merger. By analagous reasoning, the revenue of firm i'+n would also increase over any set of consumers, W2 - (d(i'+n,w) < dei' ,w) < d(j*(I' ,w) ,w)}. When w E Wl U W2, the merged parties still outcompete their outside rivals even if those rivals charge their marginal delivered cost. With respect to this set of consumers, outside firms are unaffected by the merger. Note that, unless firms i' and i'+n are neighboring firms, (i.e., i'+n - i'+l or i'-l), Wl and W2 are both empty sets.
Consider next a set of consumers that are closest to one of the merging parties, but the next-closest firm is outside the merger. For instance, let w3 E W3 - (w: dei' ,w) < d(j*(I',w),w) �~� d(i'+n,w)}. Merger between firms i' and i'+n would not affect the price for any consumer in W3 because j*(i' ,w3) - j*(I' ,w3). Gains from merger only occur when firms i' and i'+n are the two firms closest to a given consumer. Hence, there are no benefits to merger for nonneighboring firms. Notice that, with respect

to consumers in W3, outside firms are again unaffected by merger.24
From the prior discussion, outside firms receive no free-rider benefits from merger when they do not have a cost advantage. It is also apparent that no free-rider benefits occur when an outside firm does

j*(I' ,w5) cannot obtain customer w5 unless it offers the following price:

which simplifies to:
(23)
Equation (23) is identical to equation (18a), where i An outside firm with a cost advantage must set the same price before and after merger.
Based on the above discussion, each outside firm retains the same price schedule after merger. Further, this schedule still represents profit-maximizing behavior since a low-cost outside firm sets the highest possible price that still allows it to serve a given customer. Regarding consumers located where it does not possess a cost advantage, an outside firm charges its marginal delivered cost and the consumer purchases elsewhere. Thus, the following proposition holds:25
Proposition 4:
In a Bertrand-Nash equilibrium with pure price discrimination, no
free-rider problem exists. Outside firms experience no post-merger change
in profits, and their pricing strategy is unaffected by merger. Moreover,
only neighboring firms can increase their profits through merger.

The abov583r422.8oes problem 23203.85 Tm (is fits, )Tj 11.52 303.85 18 (consumer )2

can only occur �i�n�~� a more limited fashion, the free-rider benefits from merger may be small or nonexistent.
Proposi tion 4 has powerful implications. Since outside firms face unchanged competitive conditions after the merger, they are unlikely to relocate. For a similar reason, merger creates no additional incentive to enter the industry. Firms may thus receive significant long-term benefits from merger in markets characterized by

i d s

cntrrscs

incentive to refrain from merger.
Even with uniform pricing, coalitions of adequate size tend to internalize much of the benefits from coordinated behavior. The incentive to refrain from merger applies mainly to the potential formation of small coalitions under uniform pricing. Even then, firms would only hold out if they expected their rivals to merge, and some constraint limited the number of viable mergers within the market.
In spatial markets, the formation of a coalition may create two potential sources of welfare losses. First, certain consumers may be forced to buy from more distant firms. Second, if consumers can make multiple purchases (or if multiple consumers with different reservation prices are associated with each location), the associated price increases would cause a loss of allocative efficiency. 26 Coalition formation in spatial markets would often result in adverse welfare effects in the absence of any merger-specific efficiencies. However, the formation of a coalition may create welfare gains, even when the merger generates no internal efficiencies. If some consumers do not purchase from the closest source of supply, then the formation of a coalition may improve efficiency by inducing them to switch to a closer firm.
The anticompetitive aspects of merger may be tempered by the ability
of firms to eventually relocate. Depending on the nature of price competition, a number of post-merger Nash locational equilibria are possible. In the uniform-pricing case, outside firms may compete to occupy locations neighboring the merger. Hence, they would attempt to reposition

26 See Anderson and De Palma (1988), Thisse and Vives (1988), and Norman (1989) for spatial models where multiple purchases occur at each location. 21

closer to the merged parties (where prices are highest), thereby dissipating some of the gains from merger. One refinement to the set of potential locational equilibria may be to identify an equilibrium consistent with equal profits for all outside firms. In this equilibrium, firms far from the merger

less than liN from any other entity. If entry occurs at a point coincident with one of the merged parties (or nearby), the best strategy of the merger may be to drop only one location. Then, entry is not profitable.28
When firms can price discriminate, merger creates no incentive to enter the market. Merger does not affect the competitive conditions facing outside firms, and the anticipated equilibrium for prospective entrants is
also unchanged.

28 This strategy would be more likely if considerable sunk costs are
initially required to establish a brand, but fixed costs are small each period.
Note also that, in the case of a two-firm merger, entry would disrupt the collusive aspect of the merger, thereby el10.7 19.57 114 Tm 0 Tm ( (required )52 0

Appendix
A. Establishing Equation (9).
Consider the firms outside the merger. Let k' (k") denote an outside
firm that must travel kjN in a (counter)clockwise direction to reach the closest of the merged parties. When N-M is even, we can partition the set of all outside firms into two subsets, K' and K". Each subset consists of K members, (1'(1"),2'(2"), ... ,K'(K"», where K - (N-M)/2. Then, it is easy to show that if k' E K', then there exists k" E K". [When N-M is odd, this proof is slightly modified to consider the single firm that is equidistant from both merged firms.]
Note that, when evenly spaced, all outside firms face symmetric first-order conditions with respect to the prices of neighboring firms [as described by equation (3)]. Let pO' (po'') denote the price charged by the closest merged party, when that party is reached by (counter)clockwise travel. By recursive application of equation (3), we can express pk' (pk") in terms of po' (po") and pk+l' (pk+l") [relative to firm k, pk+l' (pHl") represents the price charged by the firm that is located at distance liN
in a

(A.2)
where Bn _ 4Bn - 1 - Bn -2, BO - 1, B-1 - 1(2) if N-M is even(odd).

When k - K and pO' - pO" B pO, we can solve equations (A.l) and (A.2) simultaneously to derive the following result:
pK' _ pK" _ (l/BK) [po + (BK • l)Z]
(A.3) _ 0 9.505 T236tha

B. Proof that a Pure-Strategy Equilibrium Occurs in a 2-Firm Merger When two firms merge, the equilibrium prices resulting from market-sharing strategies are described by equations (10) and (11). Accordingly, the profits of an outside neighbor are described by equation (13b):
?r1b _ [1 + (BK-l/L»)2(t/N2) - f
- 4(BK)2(t/L2N2) - f
where L _ 2BK - BK-I

(B.l)

If a neighbor instead pursues a market-appropriation strategy, it sets pI"", pO-(t/N) _ Z + [(BK-1_BK)/L) (t/N). Further, the neighbor's market segments equal 3/2N in the direction of the merger, and (BK+BK-1)/NL in the other direction. The profits from a market-appropriating strategy can thus be described as:
(B.2)

Since ?r1a < ?r1b, a neighboring firm prefers a market-sharing strategy [except when N - 3 and the lone neighbor can appropriate the merger's entire market) .
Given the equilibrium prices in equations (10) and (11), market sharing remains the preferred strategy for any other outside firm. Consider a strategy where an outside firm only appropriates the market of the neighbor nearer to the merger. Based on rival prices, its total market never exceeds 3/N. However, to appropriate that neighbor's market, pk must

be set below c+(BK-I/L) (t/N) < c+(1/3) (t/N) [since 2BK :5 BK-1 < 4BK). Thus, �~�a� < (t/N)2 - f. Referring to equation (13b), the market-sharing strategy yields higher profits. A similar argument can be used to show that an outside firm would not gain from undercutting its other neighbor.

26

Moreover, an outside firm would need to set a price below cost in order to undercut any nonneighboring firm.

C. Proof that Market Appropriation Necessarily Occurs When
pO > c + (M-1)(t/N) and ps _ pO + s(t!N).

Without loss of generality, let the border firms charge pO "" Z + a(t/N), and other coalition members charge ps - pO + s(t/N). For a given level of pO, consider the

sharing strategy:
aT (1/2(c(K)-1)2) [«M+3)/2)c2(K) - 2c(K)]
- (1/2(c(K)-1)2){[«M+3)/2)c2(K) - 2c(K)]2
- 4(c(K)-1)2c2(K»)}<l/2). (C2)
By solving the differential equation that describes Bn [refer to (9»), we can show that 2 �~� c(n) < 4. Using this result, 0 < aT < 1/2 for M �~� 4. [Since aT reaches a maximum at c(K) - 2 and M - 4, we can plug these values into equation (C2). From this result, aT < 1/2.] Hence, for any M �~� 4, market appropriation necessarily occurs when pO > c + (M-1)(t/N) - Z + (M-2)(t/N).

D. Proof that No Mixed-Strategy Equilibrium is Compatible with Sequential Limit Pricing Within the Coalition (i.e .• ps - pO + s(t/N»

Assume that the coalition uses sequential limit pricing within its borders [i.e., ps - pO + s(t/N»). Consider the mixed-strategy used by an outside neighbor, expressed as a (Borel) probability measure on
A - {pl: c �~� pl �~� a - t/2N}. Let plu represent the upper-bound of the support of this measure. Further, let F(pl) represent the corresponding cumulative density function. In response to the outside firm, the coalition chooses a probability measure with a corresponding cumulative density function, F(po). Moreover, the upper-bound of the support of the probability measure, pOu, cannot be an optimal choice unless
pOu �~� plu + (t/N). Otherwise, market appropriation would necessarily occur at pOu.
Given this reply by the coalition, we need to show that a neighboring

28

firm prefers a different strategy than F(pl). If plu �~� pOu - tiN, then plu could only be potentially optimal if it represented the single best response under a market-sharing strategy [because

Bibliography

Anderson, S.P. and DePalma, A. "Spatial Price Discrimination with Heterogeneous Products." Review of Economic Studies, Vol. 55 (1988),
pp. 573-592.

Braid, R. "Stackelberg Price Leadership in Spatial Competition," International Journal of Industrial Organization, Vol. 4 (1986),
pp. 439-449.

Caplin, A. and Nalebuff, B. "Aggregation and Imperfect Competition: On the Existence of Equilibrium." Princeton University Working Paper, 1989.

Perry, M. and Porter, R. "Oligopo1y and the Incentive for Horizontal Merger." American Economic Review, Vo1. 75 (1985), pp. 219-227.

Sa1ant, S., Switzer, S., and Reynolds, R. "Losses Due to Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium." Quarterly Journal of Economics, Vol. 48 (1983), pp. 185-200.

Salop, S.C. "Monopo1istic Competition with Outside Goods." Bell Journal of Economics, Vol. 10 (1979), pp. 141-156.

Thisse, J -F. and Vives, X. "On the Strategic Choice of Spatial Price Policy." American Economic Review, Vo1. 78 (1988), pp. 122-137.

32