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Approximating the Price E ects of Mergers: Numerical Evidence and an Empirical Application

By

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Abstract

We analyze the accuracy of rst order approximation, a method developed theoretically in Ja e and Weyl (2012) for predicting the price e ects of mergers, and provide an empirical application. Approximation is an alternative to the model-based simulations commonly employed in industrial economics. It provides predictions that are free from functional form assumptions, using data on either cost pass-through or demand curvature in the neighborhood of the initial equilibrium. Our numerical experiments indicate that approximation is more accurate than simulations that use incorrect struc-

1 Introduction

Horizontal mergers can diminish the incentives of the merging rms to compete aggressively, as each merging rm internalizes the impact of its actions on the pro ts of the other. The literature on antitrust economics characterizes this e ect as arising due to the creation of opportunity costs; each merging rm, when making a sale, forgoes with some probability a sale by the other merging rm. This interpretation is useful because these opportunity costs can be measured with data on the consumer substitution patterns and margins that arise in the pre-merger equilibrium.[1] Building on this logic, Ja e and Weyl (2012) provide general conditions under which the price e ects of mergers can be calculated, to a rst-order approximation, by multiplying these opportunity costs with an appropriate measure of cost pass-through. This calculation, hereafter referred to as \approximation," is the subject of our research.

Approximation provides an alternative to simulation for evaluating counter-factual scenarios, both in merger analysis and in industrial economics more broadly. One recognized limitation of simulation is that structural assumptions typically determine how economic behavior changes away from the initial equilibrium. In the merger context, research has shown that simulation can be sensitive to assumptions on the curvature of the consumer demand schedule (Crooke, Froeb, Tschantz, and Werden (1999)). By contrast, approximation provides robust counter-factual predictions, exploiting data on either cost pass-through or demand curvature in the neighborhood of the initial equilibrium, and allows researchers to remain agnostic about the relevant functional forms.[2]

We make two primary contributions in this paper. First, we use numerical experiments to assess the accuracy of approximation. The experiments are valuable because the theoretical results of Ja e and Weyl (2012) demonstrate the precision of approximation only with

[1] Farrell and Shapiro (2010a) refer to the opportunity costs created by a merger as gross upward pricing pressure (UPP). The Horizontal Merger Guidelines of the U.S. Department of Justice and the Federal Trade Commission, as revised in 2010, endorse upward pricing pressure as informative of the likely competitive e ects of mergers. See Horizontal Merger Guidelines 6.1:

> \The value of sales diverted to a product is equal to the number of units diverted to that product multiplied by the margin between price and incremental cost on that that product. In some cases, where su cient information is available, the Agencies assess the value of diverted sales, which can serve as a diagnostic of the upward pricing pressure.... The Agencies rely much more on the value of diverted sales than on the level of the HHI for diagnosing unilateral price e ects in markets with di erentiated products."

[2] The connection between cost pass-through and consumer demand is developed in the recent theoretical literature (e.g. Ja e and Weyl (2012), Miller, Remer, and Sheu (2012), Weyl and Fabinger (2012)).

upward pricing pressure that is arbitrarily small and with profit functions that are quadratic in price.[3] Accuracy is theoretically ambiguous outside these special cases. While it is reasonable to expect the accuracy of approximation to decrease with the magnitude of upward pricing pressure and the importance of the higher order properties of demand, it is unclear how these factors interact and at what rate the precision degrades.

We focus on horizontal mergers in the numerical experiments but note that the logic of approximation extends to other counter-factual exercises that involve perturbations to firms' marginal costs or opportunity costs. Examples include the economic impacts of emissions trading programs, gasoline taxes, tariffs and duties, and exchange rate fluctuations. Since each deals with fundamentally the same issue — the extent to which firms transmit cost shocks to final prices — our numerical experiments on mergers likely characterize the accuracy of approximation more broadly.

Our second primary contribution is an empirical application that demonstrates how approximation can be applied given scanner data with sufficient price and quantity variation. The data employed characterize unit sales and average sales prices in a consumer products industry evaluated in the past by the Antitrust Division of the U.S. Department of Justice. We use standard econometric techniques to obtain a second-order approximation of the unknown demand surface in the range of the data, which we interpret as representing the neighborhood of the pre-merger equilibrium. The results allow us to infer the appropriate measure of pass-through and apply the approximation to evaluate the likely price effects of a hypothetical merger. This approach is in stark contrast to more conventional demand estimation, which seeks to obtain the first derivatives of demand (i.e., the demand elasticities) based on functional form assumptions that restrict the second-order properties of demand.

By way of preview, the numerical results characterize the accuracy across a variety of economic environments, including a range of upward pricing pressure and four demand systems that commonly are employed in antitrust analysis: logit demand, the almost ideal demand system (AIDS) of Deaton and Muellbauer (1980), linear demand and log-linear (or isoelastic) demand. In each case, we compare approximation both to the true price effect, supplied by merger simulation conducted with the correct demand system, and to merger simulation conducted with an incorrect assumption on demand curvature.

We find that approximation provides accurate predictions when the true underlying demand schedule is linear (where it is exact) or the AIDS. Approximation is relatively less accurate when the true demand is logit or log-linear. We also find that the precision of

[3] Quadratic profit functions arise for firms with constant marginal costs and facing linear demand.

cost pass-through or the second-order properties of demand, compare approximation to merger simulation, and show that rearranging the firms' first order conditions leads to an alternative formulation of the approximation. In Section 3, we discuss the design of the numerical experiments and in Section 4 we provide the results. Finally, Section 5 develops the empirical application and Section 6 concludes.

2 Overview of Merger Approximation

2.1 Derivation and graphical illustration

We focus on models of Bertrand-Nash competition in which firms face well-behaved, twice-differentiable demand functions.[4] Each firm i produces some subset of the products available to consumers and sets prices to maximize short-run profits, taking as given the prices of its competitors. The profits of firm i have the expression

$$\pi_i = P_i^T Q_i(P) - C_i(Q_i(P)); \tag{1}$$

where P_i is a vector of firm i's prices, Q_i is a vector of firm i's sales, P is a vector containing the prices of every product, and C_i is the cost of firm i. The following first order conditions characterize firm i's profit-maximizing prices:

$$f_i(P) \equiv -\left[\frac{\partial Q_i(P)}{\partial P_i}^T\right]^{-1} Q_i(P) - (P_i - MC_i) = 0; \tag{2}$$

where MC_i is a vector of firm i's marginal costs (i.e., $MC_i = \partial C_i / \partial Q_i$). While first order conditions can be manipulated to yield various expressions, each of which characterizes the same profit-maximizing prices, the selected formulation is increasingly popular among

merger, to a first approximation, are given by the vector

$$P = \left(\frac{\partial h(P)}{\partial P}\right)^{-1}\Bigg|_{P=P^0} h(P^0):$$

Here the vector $h(P^0)$ is equivalent to net upward pricing pressure because $f(P^0) = 0$ by definition. The matrix $(\partial h(P)/\partial P)^{-1}|_{P=P_0}$ is the opposite inverse Jacobian of $h(P)$, evaluated at pre-merger prices, and captures how net upward pricing pressure is transmitted to consumers. Jaffe and Weyl (2012) refer to this matrix as *merger pass-through*. Consistent with the interpretation of upward pricing pressure as an opportunity cost, merger pass-through is related closely to the cost pass-through rates that arise in the pre-merger equilibrium. We explore this connection more deeply in Section 2.2.

To build intuition, we represent a simplified version of the approximation graphically.[6] Figure 1 plots a hypothetical function $h_i(P_i; P^0_{-i})$ for the single-product firm i, holding the prices of other products fixed at pre-merger equilibrium levels. Thus, the intersection of $h_i(P_i; P^0_{-i})$ with the horizontal axis provides the optimal price of firm i given that other prices remain unchanged from the pre-merger equilibrium.[7] The dashed line is the tangent to $h_i(P_i; P^0_{-i})$ at the pre-merger price. The post-merger price of firm i can be approximated by projecting this tangent to its point of intersection with the horizontal axis, which is equivalent to applying a single step of Newton's method. In this example, the convexity of $h_i(P_i; P^0_{-i})$ leads the approximation to understate the optimal price of the product given other prices at pre-merger levels. The convexity or concavity of the $h_i(P_i; P^0_{-i})$ depends on the higher-order properties of demand and, in general, the approximation could understate or overstate the profit-maximizing post-merger prices.

[Figure 1 about here.]

Theorem 1 implies that approximation is precise when upward pricing pressure is arbitrarily small and also with profit functions that are quadratic in price (e.g., with linear

[6] We impose that $\partial h(P)/\partial P$ is diagonal solely for the purpose of the graphical demonstration. The restriction implies that prices are unaffected by the costs of other products so that, for instance, there is no strategic complementarity or substitutability as defined by Bulow, Geanakoplos, and Klemperer (1985). Economic theory dictates that the Jacobian of $h(P)$ is never actually diagonal. Even in the case of log-linear demand

demand and constant marginal costs). Outside of these special cases, the precision of approximation is theoretically ambiguous. While the accuracy of the approximation may be expected to decrease with the magnitude of upward pricing pressure and with the curvature in $h(P)$, it is unclear how these factors interact and at what rate the precision degrades. The numerical experiments that we conduct are designed to evaluate the accuracy of approximation in such settings.

2.2 Obtaining merger pass-through

First order approximation requires knowledge of merger pass-through which, as can be ascertained from equations 2-4, depends on the first and second derivatives of demand.[8] The informational demands of approximation therefore exceed those of merger simulation, which requires knowledge only of first derivatives. In this section, we discuss how knowledge of merger pass-through can be obtained. We encourage the reader to keep in mind that the results of our numerical experiments suggest that approximation often retains precision when knowledge of merger pass-through is imperfect. Further, we develop below that pre-merger cost pass-through sometimes can serve as a reasonable proxy for merger pass-through.

One approach to obtaining the requisite demand derivatives for merger pass-through is to estimate them from data. The translog demand model of Christensen, Jorgenson, and Lau (1975) and the almost ideal demand system (AIDS) of Deaton and Muellbauer (1980) each have somewhat flexible second order properties and, given sufficient data, could be estimated. Alternatively, models with fully flexible first and second order properties could be used. Along these lines, in our empirical application we use scanner data to estimate a system of equations that provides second-order approximations to demand in the neighborhood of pre-merger equilibrium. We derive the first and second demand derivatives from the regression coefficients and apply approximation to evaluate a hypothetical merger.[9] The estimation of demand systems with flexible second order properties typically requires data with unusually rich price variation and is not feasible for many applications.

An alternative approach is to infer merger pass-through from pre-merger cost pass-through and knowledge of the first derivatives of demand. Cost pass-through has been estimated in the academic literature (e.g., Besanko, Dube, and Gupta (2005)) and in conjunction with antitrust litigation (e.g., Ashenfelter, Ashmore, Baker, and McKernan (1998)). The key to this alternative approach is that cost pass-through is tightly linked to demand

[8] We defer the derivation of merger pass-through to Appendix A.

[9] See Section 5 for details.

curvature. Following Jaffe and Weyl (2012), this connection can be derived from the first order conditions of equation 2. Consider the imposition of a per-unit tax on each product, which serves to perturb marginal costs, and denote the vector of taxes t. Since marginal costs enter quasi-linearly into the first order conditions of each firm with a coefficient of one, the post-tax pre-merger first order conditions can be written

$$f(P) + t = 0.$$

Differentiating with respect to t obtains

$$\frac{\partial P}{\partial t}\frac{\partial f(P)}{\partial P} + I = 0,$$

and algebraic manipulations then yield the pre-merger cost pass-through matrix:

$$\rho^{pre} \equiv \frac{\partial P}{\partial t} = \left(-\frac{\partial f(P)}{\partial P}\right)^{-1}. \tag{5}$$

The Jacobian of $f(P)$ depends on the first and second derivatives of demand, as is clear from equation 2.[10] Provided that the first derivatives are known, numerical optimization can be used to select second derivatives that rationalize pre-merger cost pass-through, i.e. second derivatives that minimize the "distance" between the elements in the implied opposite inverse Jacobian of $f(P)$ and the elements in the observed pre-merger cost pass-through matrix.[11] These second derivatives can then be used, in conjunction with the first derivatives, to calculate merger pass-through.

Some additional assumptions are necessary. Since the matrices that appear in equation (5) are of dimensionality $N \times N$, where N is the number of products, the relationship between pre-merger cost pass-through and the Jacobian of $f(P)$ provides N^2 equations with which to identify unknown second derivatives. An assumption that demand satisfies Slutsky symmetry is sufficient for identification in the special case of a merger among single product duopolists.[12] In other cases, second derivatives of the form $\frac{\partial^2 Q_i}{\partial P_j \partial P_k}$, for $i \neq j$, $i \neq k$ and

[10] Equation 5 clarifies the link between pre-merger cost pass-through and merger pass-through: the former depends on the Jacobian of the $f(P)$ while the latter depends on the Jacobian of $h(P)$, evaluated at pre-merger prices in both cases.

[11] In our numerical experiments, we select the second derivatives to minimize the sum of squared deviations.

[12] Slutsky symmetry implies $\frac{\partial Q_i}{\partial P_j} = \frac{\partial Q_j}{\partial P_i}$ and it follows that:

$$\frac{\partial^2 Q_i}{\partial P_j^2} = \frac{\partial}{\partial P_j}\frac{\partial Q_i}{\partial P_j} = \frac{\partial}{\partial P_j}\frac{\partial Q_j}{\partial P_i} = \frac{\partial^2 Q_j}{\partial P_j \partial P_i}.$$

$j \neq k$, are not identified from equation (5) even with Slutsky symmetry. These second derivatives are plausibly small, however, and it may be reasonable to normalize them to zero. Alternatively, Jaffe and Weyl (2012) suggest the following "horizontality" assumption on demand:

$$Q_i(P) = \left(P_i + \sum_{j \neq i} {}_j(P_j) \right); \qquad (6)$$

for some $: \mathbb{R} \to \mathbb{R}$ and $: \mathbb{R} \to \mathbb{R}$, which is sufficient for full identification. The needed second derivatives then take the form

$$\frac{\partial^2 Q_i}{\partial P_j \partial P_k}$$

Approximation differs from merger simulation primarily in how the second derivatives of demand are treated, or equivalently, in how demand elasticities are projected to change as prices move away from the pre-merger equilibrium. Whereas merger simulation employs

a clean assessment of the accuracy of approximation because it links the demand derivatives and cost pass-through that arise in pre-merger equilibrium to the underlying demand system used to conduct merger simulation.

3.2 Data generating process

3.2.1 Overview

In each experiment, we consider an industry with three single-product firms and evaluate merger between the first two firms. We begin with prices, quantities and the first firm's margin, which is sufficient information to calibrate a logit demand model. Specifically, the market shares of the first and second firms are from a uniform distribution with support between 5% and 65%. So as not to exceed the size of the market, the second firm's share also faces the upper bound of one minus the first firm's share. The third firm receives the of non-merging firms is a second-order consideration in our exercise, taking as given the upward pricing pressure created by the merger, and for the sake of simplicity we incorporate only a single non-merging firm. We note that the calibration process imposes that customer substitution among the three firms is proportional to market share for logit demand, the AIDS, linear demand and log-linear demand in the pre-merger equilibrium; the property is maintained away from the pre-merger equilibrium only for logit demand.[20] The mixed logit experiments allow us to examine more flexible consumer substitution patterns.

3.2.2 Mathematical details

We turn now to the mathematics of the selected demand systems and the calibration process. We start with the logit demand system, which takes the form

$$S_i = \frac{e^{(\xi_i - P_i)/\lambda}}{\sum_k e^{(\xi_k - P_k)/\lambda}}; \tag{13}$$

where S_i is the share of firm i (i.e. $S_i = Q_i/M$ for market size M). The unknowns include the J product-specific terms (ξ_i) and a single scaling/price coefficient (λ). The system is under-defined, which we account for by normalizing the ξ value for the last product to one. The implied elasticities evaluated at pre-merger equilibrium are

$$\epsilon_{jk} = \begin{cases} -(1 - S_j)P_j/\lambda & \text{if } j = k \\ S_k P_k/\lambda & \text{if } j \neq k \end{cases}; \tag{14}$$

is quantity. The log-linear demand system takes the form

$$\ln(Q_i) = \alpha_i + \sum_j \beta_{ij} \ln P_j ; \qquad (16)$$

where α represents the product-specific intercepts and β is as defined in equation (14). The AIDS of Deaton and Muellbauer (1980) takes the form

$$W_i = \alpha_i + \sum_j \gamma_{ij} \log P_j + \beta_i \log(X/P^*); \qquad (17)$$

where W_i is an expenditure share (i.e., $W_i = P_i Q_i / \sum_k P_k Q_k$), X is the total expenditure and P^* is a price index given by

$$\log(P^*) = \alpha_0 + \sum_k \alpha_k \log(P_k) + \frac{1}{2} \sum_k \sum_l \gamma_{kl} \log(P_k) \log(P_l):$$

We focus on the special case of $\beta_i = 0$, consistent with common practice in antitrust applications (e.g, Epstein and Rubinfeld (1999)). The restriction is equivalent to imposing an income elasticity of one. While the log-linear and linear demand systems require all the margins, the restricted AIDS model only requires two. Thus, the margins for the AIDS model are slightly different than those of the previous three models.

We also generate results for the mixed (or "random coefficients") logit demand system that is popular in empirical industrial economics research. We focus on a specific case in which market shares take the form

$$S_i = \int \frac{e^{(\xi_i - \alpha(1+\sigma\nu)P_i)/\lambda}}{\sum_k e^{(\xi_k - \alpha(1+\sigma\nu)P_k)/\lambda}} \partial F(\nu);$$

where $F(\nu)$ is a distribution that we assume to be normal with mean zero and variance one. We select α based on the already calibrated standard logit model. We select two values of σ for investigation: $\sigma = 1/(2\alpha)$, which implies that roughly 95% of consumers have downward-sloping demand, and $\sigma = 1/(4\alpha)$, which is selected as a halfway point to the standard logit model. We then we take 1,000 draws from the distribution of ν and calibrate the product specific intercepts to match the observed market shares. As in the standard logit model, we normalize the intercept of the third product to one. The results generated for this particular specification of the mixed logit model may not generalize to other specification employed in the empirical literature that feature different or multiple distributions of consumer tastes.

We nonetheless consider the exercise to have value, insofar as it shows how the accuracy of approximation can change based on the true underlying preferences of consumers.[22]

3.3 Summary statistics

Table 1 provides summary statistics on the randomly-generated industries. As shown, the average market share and margin the first firm are 37% and 46%, respectively. Substantial variation exists in each. For instance, the fifth and ninety-fifth market share percentiles are 10% and 58%. The market shares and margins of the second firm are somewhat smaller, due to the mathematical restriction that the second firm's share can never exceed one minus the first firm's share. The margins of first firm corresponds to a pre-merger own-price demand elasticities of 2.57. Given the market shares, the average implied diversion ratio from firm 1 to firm 2 is 50% in the pre-merger equilibrium and the corresponding diversion ratio from firm 2 to firm 1 is 54%. The average simulated price changes are 20%, 17%, 20% and 27% for logit demand, the AIDS, linear demand and log-linear demand, respectively, conditional on the restriction to mergers that create price effects less than 50%.[23] The randomly-drawn

3.4 Research objectives

We develop three main sets of numerical results. The first pertains to the accuracy of approximation when complete information is available either on pre-merger cost pass-through or on the second derivatives of demand in the neighborhood of pre-merger equilibrium. For each combination of draws and each demand system, we calculate approximation three ways: based on the second derivatives of demand, based on pre-merger cost through with the horizontality assumption, and based on pre-merger cost pass-through setting derivatives of the form $\partial^2 Q_i / \partial P_j \partial P_k$ equal to zero. The results characterize the performance of the approximation under the most advantageous of circumstances.

The second main set of results pertains to the accuracy of approximation when incomplete information is available on the pre-merger cost pass-through. These results may prove valuable to researchers and practitioners presented with data that are insufficiently rich to identify the full pass-through matrix. We consider two scenarios in which some of elements of the cost pass-through are known:

- Cost pass-through is available only for the merger firms.[24] To implement approximation, we impute the own-cost pass-through rate of non-merging firm using the mean of the own-cost pass-through rates of the merging firms and impute cross-cost pass-through rates involving the non-merging firms using the mean of the cross-cost pass-through rates of the merging firms.

- Only own-cost pass-through is available, i.e., the off-diagonal elements of the cost pass-through matrix are unknown. To implement approximation, we treat the cross-cost pass-through terms as equaling zero.

We also consider two scenarios in which only industry cost pass-through rates are available. Industry pass-through captures the effects of a cost shock common to all firms; from a mathematical standpoint, the industry pass-through can be calculated by summing across the rows of the cost pass-through matrix. We implement approximation two ways:

- We calculate the cost pass-through matrix that would arise given linear demand, given the the first derivatives of demand, and then scale the matrix to reproduce industry cost pass-through. We refer to this as the "adjusted-linear" method.[25]

[24] In practice, this scenario could arise when an antitrust authority has superior ability to compel document and data productions from merging firms than from non-merging firms.

[25] To obtain obtain the cost pass-through matrix, we first calculate $\partial f(P)/\partial P$ based on the equation in Appendix A, making use of the known first derivatives and presumption that the second derivatives equal

We set the own-cost pass-through rates equal to the industry pass-through rates and set the cross-cross pass-through rates to zero. This treatment is consistent with log-linear demand and we refer to it at the "log-linear" method.

Finally, we provide a number of extensions. These include (i) an examination of how well the approximation performs with mixed logit demand; (ii) an analysis of approximation for small price changes; (iii) an examination of "simple approximation" that uses pre-merger cost pass-through in place of for merger pass-through; and (iv) an analysis of approximation based on the alternative first order conditions of Section 2.4.

4 Results of Numerical Experiments

4.1 Accuracy with complete information

4.1.1 Prediction error

Table 2 summarizes the absolute prediction error of approximation that arises when complete information is available for either pre-merger cost pass-through or the second derivatives of demand in the neighborhood of pre-merger equilibrium. We define absolute prediction error as the absolute value of the difference between approximation and the true price increase. Thus, absolute error indicates the precision of approximation but not whether price predictions are overstated or understated. The table provides separate statistics for each of the posited demand systems. Observations are included in the sample only when the true price effect does not exceed 50 percent in order to provide more clarity over a reasonable range. We calculate approximations alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium ("Known Second Derivatives"); based on full knowledge of pre-merger cost pass-through and the horizontality assumption ("PTRs with Horizontality"); and based on full knowledge of pre-merger cost pass-through and the assumption that derivatives of the form $\partial^2 Q_i/\partial P_j \partial P_k$ equal zero ("PTRs with Zeros").

[Table 2 about here.]

The mean absolute prediction error (MAPE) that arises with logit demand ranges from 0.082 to 0.084. This indicates that approximation yields price predictions that are, on average, 8.2 to 8.4 percentage points different than the true price effect. Since the average

zero, and then invert following equation 5. See also Miller, Remer, and Sheu (2012), which provides an expression of $\partial f(P)/\partial P$ that is specific to linear demand.

true price effect with logit demand is 0.20, approximation is on average 41%-42% from the true effect. We explore below how that level of accuracy compares to merger simulation conducted with potentially incorrect assumptions on demand curvature. The MAPE that arises with the AIDS ranges from 0.8 to 2.6 percentage points. The average true price effect with the AIDS is 0.21 so, in our sample, approximation is on average 4.7%-15.3% from the true effect.

There is no prediction error when demand is linear. This follows from the theoretical result that approximation is exact with profit functions that are quadratic in price, as they are with linear demand and constant marginal costs. The MAPE that arises with log-linear demand and known second derivatives (in the neighborhood of pre-merger equilibrium) is 1.07. This level of prediction error is attributable to the influence of numerous “outliers” with prediction error well above two (e.g., the maximum prediction error is 66). These outliers appear to be a characteristic of the approximation, rather than a statistical quirk, in that informational setting. The MAPE that arises when the approximation is based on cost pass-through rates is 0.193 and, given the average true price effect with log-linear demand of 0.27, approximation is on average 71.5% from the true effect. The approximation does not seem to provide consistently accurate predictions under the extreme curvature of the log-linear demand system.

Figure 2 provides scatter-plots of approximation against the true price effects for logit demand, the AIDS and log-linear demand. The case of linear demand is omitted because approximation is exact in that setting. Printed on each scatter-plot is the 45-degree line; dots that appear above the line represent instances in which approximation over-predicts the true price effect while dots under the line represent under-predictions. The figure clarifies the relative accuracy of the approximation across demand systems and shows how using cost pass-through rather than direct knowledge of the second-order properties of demand (in the neighborhood of pre-merger equilibrium) does little to adversely affect accuracy. Also notable is that approximation systematically over-predicts price increases when the true underlying demand system is logit. This pattern is strongest when approximation is calculated with known second derivatives and more attenuated when approximation is calculated with cost pass-through.

[Figure 2 about here.]

4.1.2 Relative accuracy of approximation and merger simulation

Table 3 tabulates the frequency with which approximation outperforms merger simulation (in the top panel) and provides the MAPEs that arise with approximation and merger simulation (in the bottom panel). Approximation is calculated assuming full knowledge of the second demand derivatives in the neighborhood of pre-merger equilibrium. Merger simulation is conducting alternately assuming logit demand, the AIDS, and linear demand.[26] We compare approximation to each of these merger simulations when the true underlying demand system is alternately logit, the AIDS, linear and log-linear. Given the design of the experiments, merger simulation returns the true price effect only when the demand curvature assumption is correct. For example, linear demand merger simulation returns the true price effect when the true underlying demand system is linear but not when it is logit.

[Table 3 about here.]

When the true underlying demand system is logit, the approximation is more accurate than AIDS simulation in 79.1 percent of the industries considered and more accurate than linear simulation in 90.3 percent of the industries considered. When true demand is the AIDS, the approximation is more accurate than merger simulations based on logit demand and linear demand in 94.8 percent and 87.4 percent of the industries considered, respectively. The approximation always outperforms misspecified merger simulation when true demand is linear because approximation is exact in that setting. When true demand is log-linear, approximation outperforms merger simulation based on logit demand, the AIDS, and linear demand in about half the considered industries. In most cases, approximation generates smaller MAPEs than misspecified merger simulation. Together, these comparisons showcase the potential usefulness of approximation in generating robust predictions when uncertainty exists regarding the true underlying demand schedule.

4.2 Accuracy with incomplete information

4.2.1 Prediction error with incomplete information

Table 4 summarizes the absolute prediction error that arises when the second derivatives of demand are unknown and when only incomplete information is available on the pre-merger cost pass-through. Four informational scenarios are considered: pre-merger cost

[26] We exclude log-linear merger simulations because the merger simulations often do not identify any post-merger equilibrium when the true underlying demand system is logit, the AIDS or linear.

pass-through that is available only for own costs, i.e., the o -diagonal elements are unknown (\Own Cost PTRs"); pre-merger cost pass-through that is available only for the merging

linear method of allocation industry pass-through outperforms simulation when demand is the AIDS, linear or log-linear but not when demand is logit. By contrast, the log-linear method of allocating industry pass-through outperforms simulation when demand is logit. Again, since the underlying demand system would be unknown in practical applications, these tabulations do not provide clear guidance on the most appropriate treatment of industry pass-through.

4.3 Extensions

4.3.1 Accuracy with mixed logit demand

Figure 4 provides scatter-plots of approximation against the true price effects for logit and mixed logit demand systems. The approximation is calculated based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium. Two particular mixed logit models are considered, as developed in Section 3.2, based on two different price parameters:

e ect of 6.1 percentage points and depending on the precise method with approximation is conducted. The range is 3.6%-9.9% for AIDS and with log-linear it is 14%, setting aside the case of known second derivative which is again driven by outliers. Thus, under each demand system, average accuracy is improved relative to the full sample of randomly drawn industries (see Section 4.1.1). We conclude that approximation likely has enhanced usefulness when the counter-factual exercise when the perturbation to pre-merger equilibrium is less pronounced.

[Table 6 about here.]

4.3.3 Accuracy of simple approximation

In this section, we evaluate the numerical accuracy of \simple approximation," which is calculated by pre-multiw] proposed by Froeb, Tschantz, and Werden (2005) and discussed in Section 2.4. When approximation exploits known second derivatives, approximation with the baseline first order conditions is relatively more accurate for logit demand but relatively less accurate for the AIDS. This reflects, in both instances, the unexpected result that the alternative first order conditions systematically generate smaller price increases.[28] As developed above, approximation with the baseline first order conditions overstates price increases for logit demand but not (often) for the AIDS, which leads approximation with the alternative first order conditions to be accurate for logit demand and less accurate for the AIDS. A similar, though less pronounced, pattern characterizes the results when approximation exploits known cost pass-through rates and the horizontality assumption. Overall, the results indicate that neither method of approximation dominates the other in terms of accuracy, and we conclude that in some applications it could be appropriate to examine the results of both methods.

[Table 7 about here.]

5 Empirical Application

In this section, we demonstrate how the first and second derivatives of demand can be estimated and subsequently used as inputs into the J-W approximation. The data employed characterize unit sales and average sales prices in a consumer products industry evaluated in the past by the Antitrust Division. Weekly observations on four popular brands are available for more than 40 cities over roughly a four year period.[29] Our objective is to obtain a second-order approximation to the unknown demand surface over the range of the data, which we interpret as representing the neighborhood of pre-merger equilibrium. To that end, we specify the following demand system:

$$Q_i = \ _i + \sum_j \ _{ij} P_j + \sum_j \sum_{k \ j} \ _{ijk} P_j P_k + \ _i; \qquad (18)$$

where we have suppressed city and week subscripts on Q, P, and . The intercept term provides product-level fixed effects; suppressed are city and week fixed effects. This system

[28]Approximation with the alternative first order conditions generate smaller price increases in 99.5% of the logit demand industries and 100% of the AIDS industries. This also holds true for log-linear demand, where the alternative first order conditions generate smaller price increases in 93.3% of the randomly-drawn industries.

[29]

regression coefficients are unbiased provided that prices are uncorrelated with these shocks. Such as assumption would be warranted, for example, if prices are set before demand is realized in the market (e.g., see Hausman, Leonard, and Zona (1994); Weinberg and Hosken (2012)). Otherwise estimation plausibly could proceed with 2SLS, using prices in other cities/weeks as instruments, under the appropriate conditions.

Table 8 provides the demand elasticities and cost pass-through rates that are implied by the OLS regression coefficients.[33] The own-price elasticities of -3.89, -1.50, -1.56, and -2.25 imply margins for the four products of 25%, 67%, 64% and 44%, respectively. All of the cross-price elasticities are positive, consistent with consumer substitution between the products in response to price fluctuations. The own-cost pass-through rates well exceed 50% and therefore are consistent with convex demand schedules.[34] The cross-cost pass-through rates are positive, with one exception, consistent with prices being strategic complements in the sense of Bulow, Geanakoplos, and Klemperer (1985).

[Table 8 about here.]

Table 9 reports the results of approximation for a hypothetical merger of the first two products. When calculated using the baseline first order conditions and the estimated demand derivatives the predicted price changes are 36.5%, 41.1%, 27.3%, and 21.1% for the four products, respectively. Also shown are permutations based on different first order conditions and different information sets (demand derivatives versus cost pass-through) and the results of simple approximation. The advantage of these price predictions relative to merger simulation is that they make use of the estimated second-order properties of demand rather than imposing these properties through a functional form assumption — that is, they more fully allow the variation that is present in the data to inform the counter-factual predictions. While the estimation of demand systems with flexible second-order properties requires data with rich variation in prices, it is feasible that such data will become increasingly available to researchers and practitioners as firms collect, store and utilize data more efficiently.

[Table 9 about here.]

[33] We make use of equation 5 to convert the regression coefficients into cost pass-through.

[34] The implied convexity does not approach that of a log-linear demand system. In that system, the own-cost pass-through rate equals $e/(1+e)$, where e is the own-price elasticity of demand. The own-cost pass-through rates that would arise with log-linear demand, given our elasticity estimates, are 1.31, 3.00, 2.70, and 1.85, respectively, for the four products examined.

6 Discussion

Our results indicate that approximation can be a useful complement to merger simulation when sufficient data are available. Whether these complementarities are likely to be recognized by the antitrust community is unclear to us. Certainly the approximation has advantages. It provides a methodology that, in appropriate settings, can be more robust and data driven than merger simulation. Furthermore, approximation can be explained on an intuitive level simply as the product of upward pricing pressure and the appropriate measure of cost pass-through. We see the downside, relative to merger simulation, as relating primarily to economists' ability to discern cost pass-through or local demand curvature in the course of merger investigations. There is also uncertainty as to whether the derived theoretical relationship between local demand curvature and cost pass-through extends to real-world settings, or whether firms more typically apply rules-of-thumb to guide pass-through behavior. We hope that our work proves helpful to the antitrust community in identifying and evaluating these tradeoffs.

Our work also has implications for industrial economics research. In particular, one standard methodology employs model-based simulations to evaluate counter-factual scenarios that are outside the range of the available data. The structural parameters of the models typically are estimated to bring the implied first derivatives of demand close to those implied by the data. Our work highlights the importance of the *second* derivatives in driving the outcomes of simulations. Further, the numerical results we develop indicate the potential value of approximation as an alternative methodology that is applicable to some of the counter-factual scenarios of interest in industrial economics. Our results also could motivate econometric research into how to best to obtain second-order approximations to the unknown demand surface, using non-parametric regression or other techniques. The value of such research likely is enhanced by the fact that researchers increasingly have access to data with rich variation that could be exploited in estimation.

Several topics surrounding approximation remain unexplored. We provide a partial list of potential research questions here with future work in mind. First, under what theoretical conditions does approximation overstate and understate the price effects of mergers? Our numerical results indicate that approximation overstates price increases when true underlying demand schedule is logit but this relationship is ambiguous when the underlying demand schedule is instead almost ideal, log-linear or mixed logit. Research that discerns how the specific theoretical properties of these demand systems affect the performance of approximation would have value. Second, what are the most accurate ways to translate information that may be available to researchers (e.g., industry pass-through) into the cost pass-through or demand curvature information required for approximation? We have proposed a number of possibilities but have not addressed the question systematically. Finally, how accurate is approximation under different equilibrium concepts? We have focused solely on Nash-Bertrand competition but both upward pricing pressure and first order approximation are generalizable and can accommodate, for example, equilibria based on Nash-Cournot competition and consistent conjectures.

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Appendix

A Merger Pass-Through Defined

In this appendix, we provide an expression for the Jacobian of $h(P)$, which can be used to construct merger pass-through as defined by Jaffe and Weyl (2012). Using the definition $h(P) \equiv f(P) + g(P)$, we have

$$\frac{\partial h(P)}{\partial P} = \frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}. \quad (21)$$

The Jacobian of $f(P)$ can be written as:

$$\frac{\partial f(P)}{\partial P} = \begin{bmatrix} \frac{\partial f_1(P)}{\partial p_1} & \cdots & \frac{\partial f_1(P)}{\partial p_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_J(P)}{\partial p_1} & \cdots & \frac{\partial f_J(P)}{\partial p_N} \end{bmatrix}, \quad (22)$$

where N is the total number of products and J is the number of firms. The vector P includes all prices; we use lower case to refer to the prices of individual products, so that p_n represents the price of product n. In the case that product n is sold by firm i,

$$\frac{\partial f_i(P)}{\partial p_n} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} + \left(\frac{\partial Q_i^T}{\partial P_i}\right)^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}\right]^T \left(\frac{\partial Q_i^T}{\partial P_i}\right)^{-1} Q_i - \left(\frac{\partial Q_i^T}{\partial P_i}\right)^{-1} \frac{\partial Q_i}{\partial p_n}; \quad (23)$$

where Q_i and P_i are vectors representing the quantities and prices respectively of the products owned by firm i, and the initial vector of constants has a 1 in the firm-specific index of the product n. For example, if product 5 is the third product of firm 2, then the 1 will be in the 3rd index position when calculating $\partial f_2(P)/\partial p_5$. If product n is not sold by firm i, the vector of constants is $\vec{0}$, and thus

$$\frac{\partial f_i(P)}{\partial p_n} = \left(\frac{\partial Q_i^T}{\partial P_i}\right)^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}\right]^T \left(\frac{\partial Q_i^T}{\partial P_i}\right)^{-1} Q_i - \left(\frac{\partial Q_i^T}{\partial P_i}\right)^{-1} \frac{\partial Q_i}{\partial p_n}. \quad (24)$$

The matrix $\partial g(P)/\partial P$ can be decomposed in a similar manner:

$$\frac{\partial g(P)}{\partial P} = \begin{bmatrix} \frac{\partial g_1(P)}{\partial p_1} & \cdots & \frac{\partial g_1(P)}{\partial p_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_K(P)}{\partial p_1} & \cdots & \frac{\partial g_K(P)}{\partial p_N} \\ 0 & \cdots & 0 \end{bmatrix}; \qquad (25)$$

where N is the number of products and K is the number of merging firms. Notice that $\partial g(P)/\partial P$ includes zeros for non-merging firms, because the merger does not create opportunity costs for these firms. In the case that product n is sold by a firm merging with firm i (this does not include firm i itself),

$$\frac{\partial g_i(P)}{\partial p_n} = \left(\frac{\partial Q_k^T}{\partial P_i}\right)^{-1} \frac{\partial Q_k^T}{\partial P_i} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \qquad (26)$$

$$+ \left(\frac{\partial Q_k^T}{\partial P_i}\right)^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}\right]^T \left(\frac{\partial Q_k^T}{\partial P_i}\right)^{-1} \frac{\partial Q_k^T}{\partial P_i} \left(\frac{\partial Q_k^T}{\partial P_i}\right)^{-1} \left[\frac{\partial^2 Q_j}{\partial P_i \partial p_n}\right]^T (P_j - C_j);$$

where Q_j, P_j, and C_j are vectors of the quantities, prices, and marginal costs respectively of products sold by firms merging with firm i, and the vector of 1s and 0s has a 1 in the merging firm's firm-specific index of the product n. For example, if product 5 is the third product of firm 2, and firm 2 is merging with firm 1, then the 1 will be in the 3rd index position when calculating $\partial g_1(P)/\partial p_5$. It is an important distinction that – supposing there are more than two merging parties – the index j refers to all of the merging parties' products, excluding firm i's products. If product n is not sold by any firm merging with firm i (including a product sold by firm i),

$$\frac{\partial g_i(P)}{}$$

Figure 1: Graphical Illustration of First Order Approximation

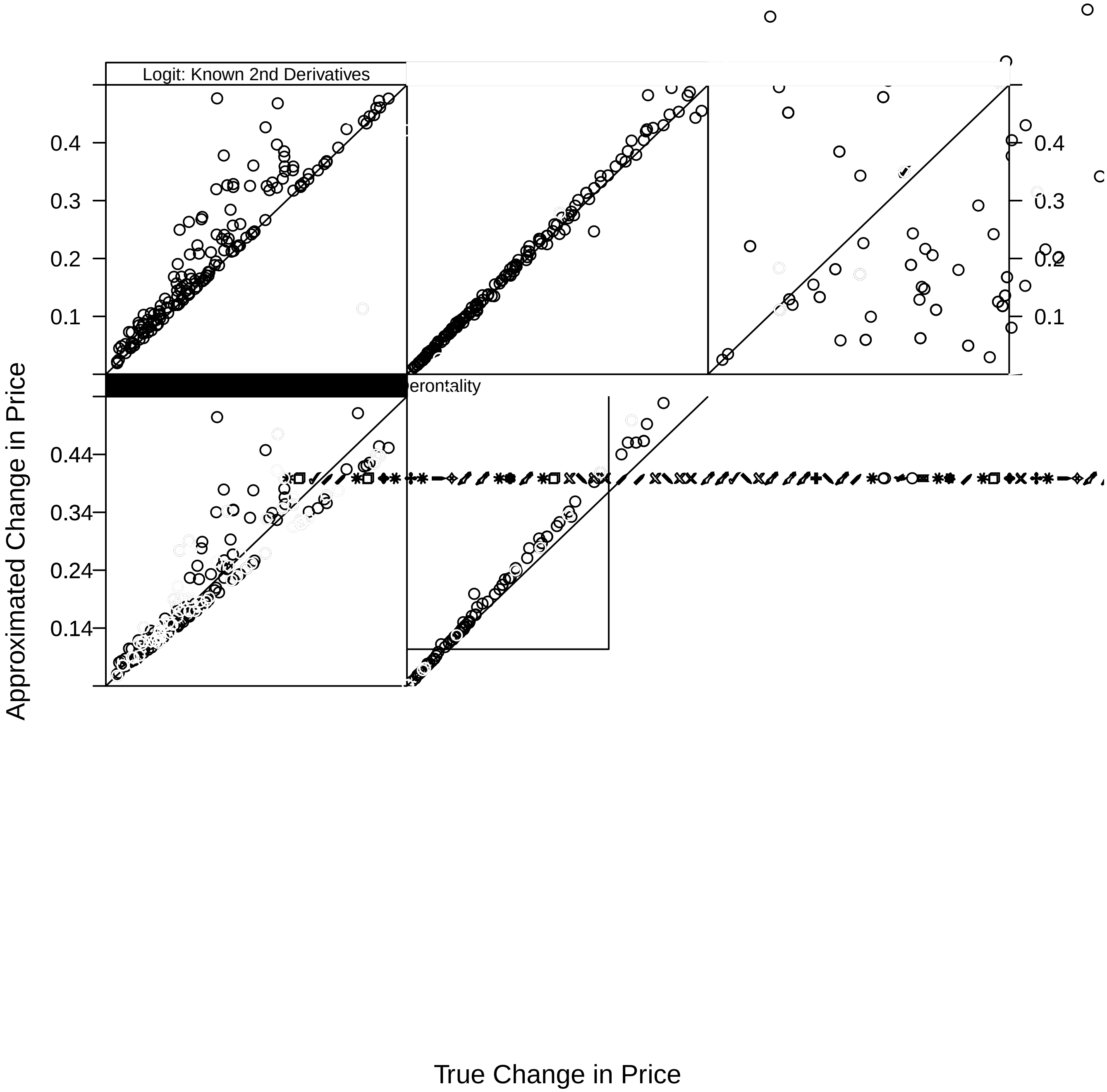

Figure 2: Prediction Error with Complete Information.

Notes: The gure provides scatter-plots of approximation against the true price e ect for logit demand, the AIDS and log-linear demand. The case of linear demand is omitted because approximation is exact in that setting. Approximations are calculated alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium (\Known 2nd Derivatives"); based on full knowledge of pre-merger cost pass-through and the horizontality assumption (\PTRs with Horizontality"); and based on full knowledge of pre-merger cost pass-through and the assumption that derivatives of the form $\partial^2 Q_i/\partial P_j \partial P_k$ equal zero (\PTRs with Zeros").

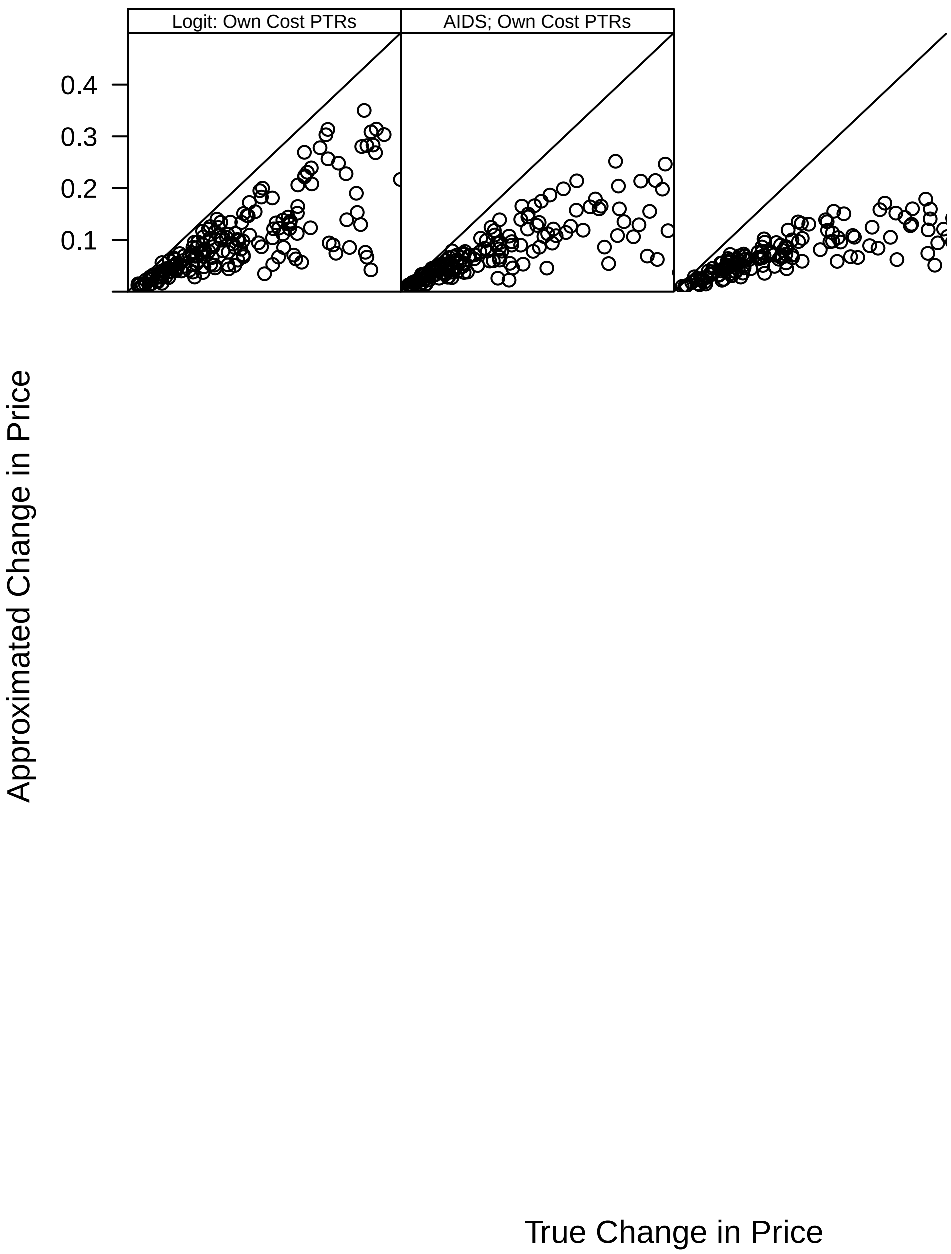

Figure 3: Prediction Error with Incomplete Information

Notes: The gure provides scatter-plots of approximation against the true price e ect for logit demand, the AIDS, linear demand and log-linear demand. Four informational scenarios are considered: pre-merger cost pass-through that is available only for own costs (\Own Cost PTRs"); pre-merger cost pass-through that is available only for the merging rms (\Merging

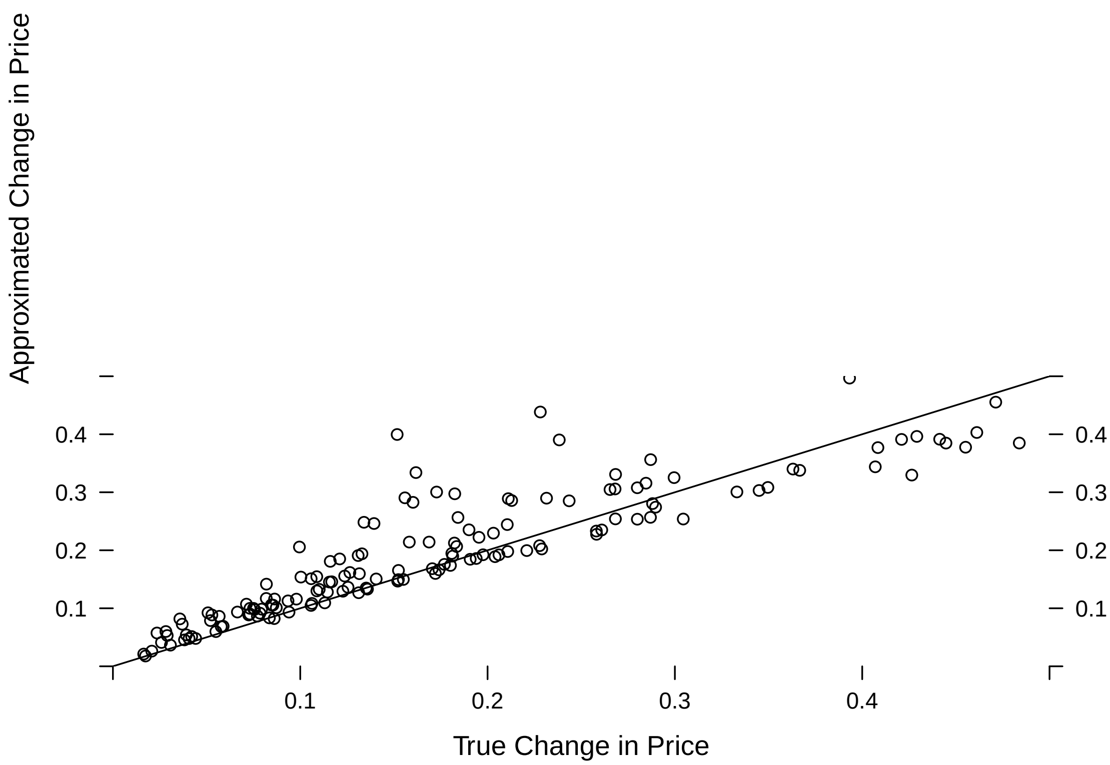

Figure 4: Prediction Error with Complete Information { Logit and Mixed Logit Demand
Notes: The gure provides scatter-plots of approximation against the true price e ect for logit and mixed logit demand. The approximation is calculated based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium.

Figure 5: Prediction Error with Simple Approximation

Notes: The figure provides scatter-plots of simple approximation against the true price effect for logit demand, the AIDS, linear demand and log-linear demand. The approximation is calculated based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium.

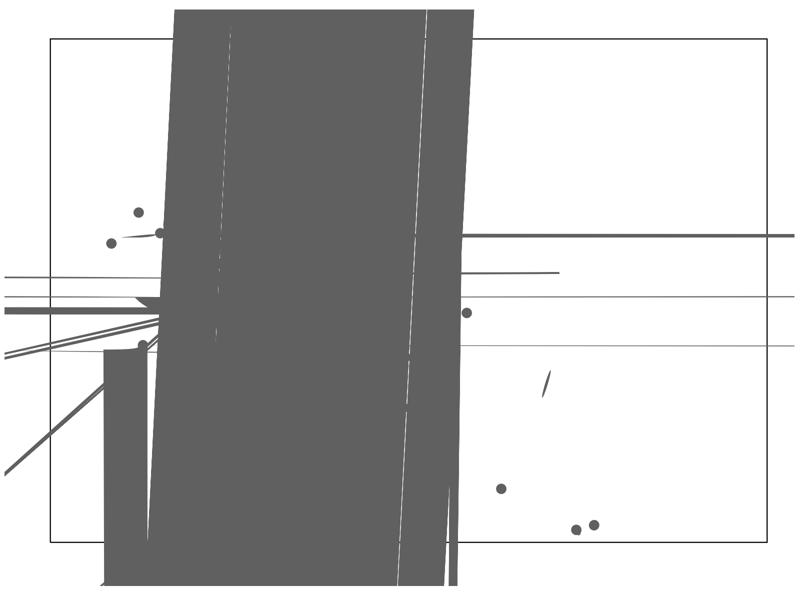

Figure 6: Prices and Unit Sales in a Representative City.

Notes: The gure provides a scatter-plot of the weekly average sales price and unit sales for one product in a representative city. To protect the con dentiality of the data, a small number of outliers have been omitted and both average sales price and unit sales have been scaled by an unspeci ed constant and perturbed additively by a uniformly distributed random variable.

Table 1: Summary Statistics

	Mean	St. Dev.	5th pctile	95th pctile
Characteristics of Firm 1				
Market share	0.37	0.15	0.10	0.58
Margin	0.46	0.17	0.22	0.75
Own-price elasticity	2.57	1.48	1.33	4.52
Characteristics of Firm 2				
Market share	0.31	0.15	0.08	0.54
Margin	0.44	0.21	0.17	0.87
Own-price elasticity	2.86	1.83	1.15	5.96
Consumer Substitution				
Diversion from 1 to 2	0.50	0.23	0.15	0.90
Diversion from 2 to 1	0.54	0.22	0.16	0.90
Merger Simulation Results Conditional on PC < 0.50				
Logit demand	0.20	0.13	0.04	0.44
AIDS	0.17	0.13	0.02	0.43
Linear demand	0.20	0.13	0.04	0.46
Log-linear demand	0.27	0.13	0.06	0.47
Mixed Logit demand ($\frac{}{4}$)	0.20	0.13	0.04	0.44
Mixed Logit demand ($\frac{}{2}$)	0.19	0.12	0.03	0.44

Notes: Summary statistics are based on 300 randomly-drawn industries. The merger simulation results show changes in rm 1's price, conditional on that change being under 50%. With logit demand, 242 of the 300 randomly-drawn industries produce such a price change. With the AIDS, linear demand, and log-linear demand, 191, 190, and 45 industries produce such a price change, respectively.

Table 2: Absolute Prediction Error with Complete Information

	Logit Demand			AIDS		
	Mean	5th pctile	95th pctile	Mean	5th pctile	95th pctile
Known Second Derivatives	0.084	0.000	0.339	0.008	0.000	0.023
PTRs with Horizontality	0.082	0.001	0.319	0.026	0.001	0.081
PTRs with Zeros	0.083	0.002	0.320	0.013	0.000	0.046
	Linear Demand			**Log-linear Demand**		
	Mean	5th pctile	95th pctile	Mean	5th pctile	95th pctile
Known Second Derivatives	0	0	0	1.072	0.003	2.189
PTRs with Horizontality	0	0	0	0.193	0.006	0.395
PTRs with Zeros	0	0	0	0.193	0.006	0.395

Notes: The table provides summary statistics regarding the absolute prediction errors of approximation. Absolute prediction error is de ned by the absolute value teetprob approximatthete

Table 3: Approximation with Complete Information Versus Merger Simulation

Panel A: Frequency with which Approximation Outperforms Simulation

Table 4: Absolute Prediction Errors with Incomplete Information

	Logit Demand			AIDS		
	Mean	5th pctile	95th pctile	Mean	5th pctile	95th pctile
Own Cost PTRs	0.096	0.013	0.260	0.094	0.007	0.288
Merging Firms' PTRs	0.019	0.001	0.058	0.018	0.002	0.071
Ind. PTRs { Adj.-Linear	0.266	0.003	1.107	0.025	0.001	0.106
Ind. PTRs { Log-Linear	0.052	0.005	0.169	0.067	0.001	0.246
	Linear Demand			Log-linear Demand		
	Mean	5th pctile	95th pctile	Mean	5th pctile	95th pctile
Own Cost PTRs	0.128	0.017	0.330	0.193	0.006	0.395
Merging Firms' PTRs	0.025	0.003	0.080	0.193	0.006	0.395
Ind. PTRs { Adj.-Linear	0	0	0	0.142	0.007	0.320
Ind. PTRs { Log-Linear	0.078	0.004	0.259	0.193	0.006	0.395

Notes: The table provides summary statistics regarding the absolute prediction errors of approximation. Separate statistics are shown for logit demand, the AIDS, linear demand and log-linear demand. Observations are included only when the true price e ect does not exceed 50 percent. Four informational scenarios are considered: pre-merger cost pass-through that is available only for own costs, i.e., the o -diagonal elements are unknown (\Own Cost PTRs"); pre-merger cost pass-through that is available only for the merging rms (\Merging Firms' PTRs"); industry cost pass-through that is apportioned using the adjusted-linear method (\Ind. PTRs { Adj.-Linear"); and industry cost pass-through that is apportioned using the log-linear method (\Ind. PTRs { Log-Linear").

Table 6: Mean Absolute Prediction Error for Small Price Changes

	Logit	AIDS	Linear	Log-Linear
Known Second Derivatives	0.016	0.002	0	4.614
PTRs with Horizontality	0.010	0.005	0	0.091
PTRs with Zeros	0.010	0.004	0	0.091

Notes: The table provides the mean absolute prediction errors of approximation that arise when the true price e ect does not exceed 10%. Separate statistics are shown for logit demand, the AIDS, linear demand and log-linear demand. The approximation is calculated alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium (\Known Second Derivatives"); based on full knowledge of pre-merger cost pass-through and the horizontality assumption (\PTRs

Table 7: Mean Absolute Prediction Error with Alternative FOCs

	Logit	AIDS	Linear	Log-Linear
Known Second Derivatives				
Baseline FOC	0.084	0.008	0	1.072
Alternative FOC	0.061	0.103	0	0.150
PTRs with Horizontality				
Baseline FOC	0.082	0.026	0	0.193
Alternative FOC	0.084	0.054	0	0.176

Notes: The table provides the mean absolute prediction errors that arise with both the baseline first order conditions and with the alternative first order conditions. Separate statistics are shown for logit demand, the AIDS, linear demand and log-linear demand. Observations are included only when the true price effect does not exceed 50 percent. The approximation is calculated alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium ("Known Second Derivatives") and based on full knowledge of pre-merger cost pass-through with the horizontality assumption("PTRs with Horizontality").

Table 8: Results from OLS Regressions

Panel A: Demand Elasticity Estimates

	Product 1	Product 2	Product 3	Product 4
Product 1	-4.22	0.19	0.09	0.55
Product 2	1.96	-1.50	0.35	1.78
Product 3	1.16	0.34	-1.59	0.65
Product 4	1.88	0.81	0.37	-2.17

Panel B: Cost Pass-Through Estimates

	Product 1	Product 2	Product 3	Product 4
Product 1	0.82	0.17	-0.07	0.31
Product 2	0.64	1.32	0.08	1.65
Product 3	1.14	0.52	2.54	3.75
Product 4	0.35	0.29	0.03	1.36

Notes: The elasticities and cost pass-through rates are inferred from OLS regression coe cients. In Panel A, the top number in the second column is the elasticity of demand for product 1 with respect to the price of product 2, and the remaining numbers are calculated accordingly. In Panel B, the top number in the second column is the pass-through rate of product 1 with respect to the costs of product 2, and again the remaining numbers are calculated accordingly.

Table 9: Approximation Results for Merger of Products 1 and 2

	Product 1	Product 2	Product 3	Product 4
Known 2nd Derivatives				
Baseline FOC	36.5%	41.1%	27.3%	21.1%
Alternative FOC	28.5%	31.0%	21.2%	16.2%
PTRs with Horizontality				
Baseline FOC	57.0%	51.1%	40.9%	29.7%
Alternative FOC	37.3%	32.8%	26.7%	19.3%
PTRs with Zeros				
Baseline FOC	41.1%	36.5%	29.4%	21.3%
Alternative FOC	29.8%	26.0%	21.2%	15.3%
Simple Approximation	26.0%	20.3%	18.2%	12.8%

Notes: Approximation is based on the estimated demand derivatives and either uses these derivatives directly (\Known 2nd Derivatives") or uses the implied cost pass-through rate matrix.