Dynamic Price Competition:
Theory and Evidence from Airline Markets*

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Abstract

We estimate welfare effects of dynamic price competition in the airline industry. To do so, we introduce a general dynamic pricing game where sellers are endowed with finite capacities and face uncertain demands toward a sales deadline. We establish sufficient conditions for equilibrium existence and uniqueness, and for convergence to a system of differential equations. With the equilibrium characterization and comprehensive pricing and bookings data for competing airlines, we estimate that dynamic pricing results in higher output but lower welfare than under uniform pricing.

1 Introduction

Dynamic pricing is commonly used by firms selling fixed inventory by a set deadline. Examples range from seats on airlines and trains, tickets for entertainment events, to reservations for cruises, and inventory in retailing. In these markets, capacity influences prices in important ways. First, prices adjust as the opportunity cost of selling changes with

Our dynamic pricing game extends earlier single-firm frameworks (Gallego and Van Ryzin, 1994; Zhao and Zheng, 2000; Talluri and Van Ryzin, 2004) to oligopoly.[1] In the main text we focus on a duopoly where each firm offers a single product. In the appendix, we extend our results to an arbitrary number of firms, each offering an arbitrary number of products. Each firm is exogenously endowed with limited initial capacity that must be sold by a deadline.[2] After the deadline has passed, unsold capacities are scrapped with zero value. Firms are not allowed to oversell. Products are imperfect substitutes and satisfy general regularity conditions. Consumers arrive randomly according to time-varying arrival rates with time-varying preferences. Each consumer is short-lived and decides whether to purchase an available product or select an outside option. Our demand assumptions are motivated by recent empirical evidence (Hortaçsu et al., 2021b). In every period, firms simultaneously choose prices after observing remaining capacities for all products; demand is realized, capacity constraints are updated, and the process repeats until the perishability date or until all products are sold out. We call this game the *benchmark model*.

Our model produces a rich set of equilibrium strategies because competitor prices affect both current demand and opportunity costs of remaining capacity. This can create incentives to offer fire sales as in Dilme and Li (2019), where a single firm competes with its future self for forward-looking buyers.[3] However, a firm might also want to charge a high price in order to drive the competitor to sell out as in Martínez-de Albéniz and Talluri (2011), where firms offer perfect substitutes.[4]

through Poisson arrivals, and preferences are modeled through discrete choice nested logit demand. We use search data for one airline to inform arrival process parameters that are then scaled up to account for unobserved searches, e.g., via online travel agencies or a competitor's website. In total, we estimate demand for 58 duopoly routes. We nd sig-ni cant variation in willingness to pay across routes and across days from departure for a given route. In general, demand becomes more inelastic as the departure date approaches. Average own-price elasticities are -1.4.

With the demand estimates, we simulate equilibrium market outcomes using the differential equation characterization. This allows us to recover the own/competitor scarcity effects and rm strategies for all potential states—some games (route-departure dates) feature over 131 million potential states. We nd that overwhelmingly (but not all) of the realized stage games are of strategic complements.

We compare market outcomes of dynamic pricing to uniform pricing where each rm commits to a single price for each ight over time. We nd the opposite welfare effect compared to earlier analyses, including Hendel and Nevo (2013) in retailing, Castillo (2020) in ride-share, and Williams (2022) for single-carrier airline markets, in that dynamic pricing expands output but lowers total welfare compared to uniform pricing. This occurs because dynamic pricing softens price competition toward the departure date, despite fea-

2 Model of Dynamic Price Competition

We begin by detailing the demand assumptions that we use in our analysis in Section 2.1. Our exposition of demand is for an arbitrary number of products. In Section 2.2 we intro-

iii) Differentiability and diagonally dominant Jacobi matrix: *For all subsets* $A \quad J$ *and* $j \in A$, $s_j(p^A;\ \ ,A)$ *is smooth in* *and* p^A, *and*

$$\frac{\partial s_j}{\partial p_j}(p^A;\ \ ,A) > \sum_{j' \in A \setminus \{j\}} \frac{\partial s_{j'}}{\partial p_j}(p^A;\ \ ,A); \qquad (1)$$

Furthermore, for any subset $A \quad J$ *there exists a* $C > 0$ *such that for all* p^A

$$|s_j(p^A;\ \ ,A)| < C \left(\frac{\partial s_j}{\partial p_j}(p^A;\ \ ,A) \quad \sum_{j' \in A \setminus \{j\}} \frac{\partial s_{j'}}{\partial p_j}(p^A;\ \ ,A) \right). \qquad (2)$$

Assumption 1-i) ensures that the demand system is well-defined when products sell out. The first condition in Assumption 1-iii) (Equation 1) ensures that the Jacobi matrix $D_p s(p)$ is non-singular by the Levy-Desplanques Theorem (see e.g. Theorem 6.1.10. in Horn and Johnson (2012)). This condition intuitively means that a price change of product j should impact demand of product j more than it impacts the sum of demand of all other products. The second condition in Assumption 1-iii) (Equation 2) ensures that

A

Recall that Assumption 1 guarantees that $\max_{p \in \mathbb{R}^{A}} s(p)^{\top}(p - c)$ has an interior solution. Assumption 2 guarantees that the system of first-order conditions of this problem has a unique solution. Together, these assumptions replace the assumption of log-concavity commonly made in single-product, single-firm setting.

We now omit the conditioning arguments and A in demand and demand elasticities whenever the meaning is unambiguous. When the time index is relevant, we write $s_{j,t}(p) := s_j(p; {}_t, A_t)$. Further, we let the probability of choosing the outside option be equal to $s_{0,t}(p) := 1 - \sum_{j \in J} s_{j,t}(p)$.

2.1.1 Parametric Demand Models

We illustrate theoretical insights with a simple logit demand specification, i.e.,

$$s_{j,t}(p) = \frac{\exp\left(\frac{{}_j - {}_t p_j}{}\right)}{1 + \sum_{j' \in A_t} \exp\left(\frac{{}_{j'} - {}_t p_{j'}}{}\right)}. \tag{3}$$

We set $_t = _t$ so that $_t =$ is the time-variant marginal utility to income, and > 0 is a scaling factor. The parameter $_j =$ is the product-specific value of product j. Note that when $\to 0$, competition collapses to standard Bertrand. As $\to \infty$, products become perfectly differentiated. In our empirical analysis, we consider the more flexible nested logit demand model. Both classic logit and nested logit demand functions satisfy Assumptions 1 and 2 (see Appendix C).

2.2 Single Firm Model

We first discuss a single firm, multi-product dynamic pricing model with two goals in mind. The first is to introduce supply-side notation that we carry over to the competitive model. The second is to showcase that the single-firm problem is well behaved and exhibits nice properties. All of them fail in the oligopoly model.

A single firm M offers J products for sale with an initial inventory $K_{j,0} \in \mathbb{N}$ of each product j. We do not model the initial capacity choice. Let K_t

as $\;\to 0$, *which satisfies*

$$\dot{\ }_{M,t}(K) = \ _t \max_p \sum_{j \in J} s_{j,t}(p)\left[p_j \ _{M,t}(K) \ _{M,t}(K - e_j)\right]$$

with boundary conditions (i) $\ _{M,T}(K) = 0$ *for all* K, *(ii)* $\ _{M,t}(0) = 0$ *for all* t, *and* $\ _{M,t}(K) = -\infty$ *if* $K_j < 0$ *for a* $j \in J$ *.*

Given a capacity vector K, corresponding available products $A = \{j : K_j \neq 0\}$, and the vector of opportunity costs $\omega_{M,t}(K)$ of products $j \in A$, the first-order condition for profit-maximizing prices $p_{M,t}(K) \in \mathbb{R}^A$ can be written in matrix form,

$$p_{M,t}(K) = \underbrace{\omega_{M,t}(K)}_{\text{opportunity costs}} \underbrace{\left[D_p s_t(p_{M,t}(K))\right]^{-1} s_t(p_{M,t}(K))}_{=\hat{\ }(p;\ ,A)\ \text{inverse quasi own-price elasticities}} . \tag{4}$$

Hence, the pricing policy $p_{M,t}(K)$ is continuous in time and well behaved. The evolution of the price vector $p_{M,t}(K_t)$ is then governed by the evolution of the random variable representing the opportunity costs and quasi-price elasticities of demand. The following proposition summarizes well-known properties of an optimal control problem, including monotonicity and concavity of the value function in the capacity vector. We also derive properties of the stochastic process governing the opportunity costs $\omega_{j,t}(K_t)$.

Proposition 1. *The solution to the continuous-time single-firm revenue maximization problem in Lemma 1 satisfies the following:*

i) $\ _{M,t}(K)$ *is de 8.including*

terminology.

Definition 1. We say that a *competitor's sale intensifies competition* in a state (K, t) if $\omega^{f}_{f',t}(K) > 0$ and that a *competitor's sale softens competition* in a state (K, t) if $\omega^{f}_{f',t}(K) < 0$.

For a stage game with $\omega^{f}_{f',t} \neq 0$, we cannot apply results from Caplin and Nalebuff (1991) or Nocke and Schutz (2018). Payoffs are also neither super-modular nor log-supermodular (Milgrom and Roberts, 1990), and the stage game is also not a potential game. In the next section, we derive conditions on the stage game that guarantee uniqueness of equilibrium outcomes to show in how far Lemma 1 generalizes to a duopoly.

3 Analysis of the Duopoly Model

In this section, we derive theoretical properties of the dynamic pricing game. We start with an analysis of uniqueness and continuity of stage game equilibria, which allows us to generalize Lemma 1. We also provide additional theoretical insights on competition, the role of capacity, and pricing dynamics.

3.1 Equilibrium Existence, Uniqueness, and Continuity

3.1.1 Sufficient Condition for Equilibrium Uniqueness in the Stage Game

We consider the stage game for an arbitrary matrix of opportunity costs ω. We drop the time index and capacity argument in all expressions temporarily. Our first result presents sufficient conditions for existence and uniqueness of an equilibrium of the stage game. Recall that the best responses of both firms are uniformly bounded by Assumption 1-(iii) and hence, must satisfy a first-order condition. We can write the first-order condition of firm f's profit maximization problem as

$$g_f(p) = p_f,$$

where

$$g_f(p) := \underbrace{-\left(\omega_f^f + \frac{\frac{\partial s_{f'}}{\partial p_f}(p)}{\frac{\partial s_f}{\partial p_f}(p)}\,\omega_f^{f'}\right)}_{\substack{\text{net opportunity cost}\\ \text{of selling}}} \underbrace{-\, s_f(p)\left(\frac{\partial s_f(p)}{\partial p_f}\right)^{-1}}_{\substack{\text{inverse quasi}\\ \text{own-price elasticity}}}. \tag{6}$$

By Kellogg (1976),[10] the following assumption then guarantees that there is a unique solution to this system of equations.

Assumption 3. *Suppose the following two conditions hold:*

i) $\frac{\partial g_f}{\partial p_f}(p) - 1 \neq 0$ *for all* p *and* $f = 1, 2$*;*

ii) $\det\left(D_p\, g(p) - I\right) \neq 0$ *for all* p*, where* $g(p) := (g_1(p), g_2(p))$*.*

To better understand Assumption 3, first note that with a single firm, the assumption guarantees that the first-order condition of the firm is either increasing or decreasing everywhere in its price. Assumption 3-(i) is always satisfied for demand functions that are log-concave in each dimension. Mathematically, Assumption 3-(ii) is related to Assumption 2, but the inverse quasi-own price elasticity is replaced by the function $g(p)$. If the competitor scarcity effect is zero, one can see from Equation (6) that Assumption 2 implies Assumption 3. If the competitor scarcity effect is not zero, the first-order condition is more complex than in the single-firm setting since the net opportunity cost of selling depends on the ratio of derivatives of the demand of the two firms.

Lemma 2. *Let Assumptions 1, 2 and 3 hold. Then, the stage game admits a unique equilibrium.*

Note that Lemma 2 establishes uniqueness and existence simultaneously. Under the commonly made assumption of independence of irrelevant alternatives (IIA) that is satisfied uniqueness for arbitrary stage games. In the next subsection we provide an example for that yield multiple equilibria.

3.1.2 Continuity of Equilibrium Prices in Scarcity Effect Matrix

Next, we study the stage game parameterized by scarcity effects and demand parameters . We show that if and remain in a compact neighborhood in which the stage game admits a unique solution, then equilibrium prices denoted by p (,) are continuous in and . Consequently, a small change in the opportunity costs does not change prices substantially. In the dynamic game, as long as no sales occur, prices do not jump over time provided and stay in the compact neighborhood. This property turns out useful for generalizing Lemma 1 and simulating equilibrium price paths.

Lemma 3. *Let Assumptions 1 and 2 hold. If the equilibrium of the stage game is unique for a compact set of* (,) $\in \mathcal{O}$*, then there exists an equilibrium price vector* p (,) *for any* (,) *such that* p (,) *is continuous in* (,) *on* $\mathcal{O}$*.*

Given Assumption 2, Assumption 3-ii) is satisfied for any matrix of scarcity effects in a neighborhood $\mathcal{O}$ that contains the zero matrix $= 0$ by continuity. However, Assumption 3-ii) can fail for non-zero values of scarcity effects. In such cases, we can get multiplicities of equilibria that can potentially result in price jumps that are not caused by a change in inventory in the dynamic game. The following discussion illustrates this point.

Lemma 3 can fail if Assumption 3 is violated. To see this, consider logit demand such that $_1 = \ _2 = 0$, and $= 1$. In this case, Assumption 3 is equivalent to

$$\left(s_1(p) + \omega_2^1 s_0(p)\right)\left(s_2(p) + \omega_1^2 s_0(p)\right) \le 1 + \frac{1 - s_1(p) - s_2(p)}{s_1(p)s_2(p)}.$$

Note that this condition does not depend on the firms' own-product scarcity effects ω_1^1 and ω_2^2. Therefore, we set own-product scarcity effects equal to zero and parameterize competitor scarcity effects using a continuous function. We plot the parameterization of (ω_1^2, ω_2^1) in Figure 1-(a). We plot the corresponding equilibrium prices for both firms in

1-(b). The figure shows that multiplicity of equilibria can occur and there are jumps in prices—even when scarcity effects change continuously.

Figure 1: Multiplicities in stage-game equilibria

(a) Parametrization of $(\omega_2^1(x), \omega_1^2(x))$ (b) Multiplicity in equilibrium prices

Note: In these graphics we parameterize (ω_1^2, ω_2^1) with a curve $(\omega_2^1(x), \omega_1^2(x)) = (15\cos(\frac{\pi}{2}x), 15\sin(\frac{\pi}{2}x))$, $x \in [0,1]$, where we set $(\omega_1^1, \omega_2^2) = (0,0)$, and assume logit demand with $= (1,1)$, $_t = 1$ and scaling factor $= 1$. Panel (a) depicts the parameterized curve and panel (b) equilibrium prices of both firms given (ω_1^2, ω_2^1) at varying values of x.

3.1.3 Characterization of Continuous-time Limit

Using Lemma 3 and Lemma 5 in the appendix, we can generalize Lemma 1 to a duopoly as long as the time horizon is not too long. We state the result formally below. The equilibrium characterization is useful because it allows us to simulate equilibrium outcomes in our empirical analysis for high-dimensional games.

Proposition 2 (Continuous-time Limit) Let Assumptions 1, 2, and 3 hold for $= 0$. For every K, there exists a $T_0(K) > 0$, non-increasing in (K hold for *where* $f' \neq f$, *with boundary conditions (i)* $_{f,T}(K) = 0$ *for all* K*, (ii)* $_{f,t}(K) = 0$ *if*

3.2 Additional Theoretical Results on Dynamic Price Competition

3.2.1 Prices as Strategic Substitutes vs Strategic Complements

In a static Bertrand game with imperfect substitutes, prices are strategic complements for commonly used demand specifications, including logit and nested logit demand systems. Hence, competition unambiguously lowers prices. Due to the presence of competitor scarcity effects, our model results in pricing games that may be strategic substitutes or strategic complements, even for a simple demand systems.

In order to understand the strategic incentives when a competitor changes its price, recall that the first derivative with respect to p_f of firm f's payoff function is given by $p_f \mapsto \frac{\partial s_f}{\partial p_f}\left(p_f - g_f(p)\right)$. By Assumption 1-ii), the first-order condition is satisfied if and only if $p_f = g_f(p)$. Furthermore, by Assumptions 1-i) and 3-i), there is a unique interior maximum of firm f's payoff function for any competitor price $p_{f'}$ and $g_f(p)$ is strictly decreasing.

How does firm f's best-response change if the competitor raises its price? Firm f's best response increases, i.e., the competitor's price is a strategic complement, if $\frac{\partial g_f}{\partial p_{f'}} > 0$ and it decreases, i.e., the competitor's price is a strategic substitute, if $\frac{\partial g_f}{\partial p_{f'}} < 0$. Typically, the literature assumes monotonicity of the own-price elasticity in the competitor's price, which is, for example, guaranteed for log-concave demands. In this case, prices are strategic complements. However, in our setting, the strategic forces are less straightforward du4.3462 Tuu46(straight

Proposition 3. *Let* $_t \quad , \quad _t \quad$. *Then, for* K *with* $\underline{K} := \min_f K_f$, *the following holds:*

$$p_{f,t}(K) = p_{f,T} + O(|T \quad t|^{\underline{K}}), \; t \to T \quad \textit{for } f = 1,2,$$

i.e., price changes close to the deadline are at most of order $\underline{K}$. *If* $\lim_{t \to 0} \frac{\partial^{\underline{K}}}{(\partial t)^{\underline{K}}} \quad _{h,t}(K \quad e_h) \neq 0$ *for all* f *with* $K_h = \underline{K}$, *then*[13]

$$p_{f,t}(K) = p_{f,T} + \quad (|T \quad t|^{\underline{K}}), \; t \to T \quad \textit{for } f = 1,2,$$

i.e., price changes are exactly of order $\underline{K}$.

We illustrate these price competition effects in Figure 14 in Appendix D. We consider firms with K =(5,4); (4,4); and (3,4) capacities. Note that (4,4) prices are the lowest, and the firm with the lowest capacity XX[14]

Empirically, this means that we expect firms to benefit from dynamic pricing whenever remaining capacities are distributed unequally across firms, as equally distributed capacities result in intense price competition.

3.2.3 Independence of Irrelevant Alternatives and Markup Formula

Finally, we show that for demand specifications that satisfy the commonly used assumption of "Independence of Irrelevant Alternatives (IIA)," the stage game admits an equilibrium for any scarcity matrix . Moreover, the game satisfies a markup formula.

Assumption 4 (Independence of Irrelevant Alternatives (IIA)).

Proposition 4 implies that equilibrium prices p (,) satisfy a markup formula

$$\frac{p_f(\ ,\ )\quad c_f(p_{f\,0};\ ,\ )}{p_f} = \quad \frac{1}{\ _f(p)}, \tag{8}$$

where $\ _f(p) = \frac{@\, s_f(p)}{@\, p_f}\frac{p_f}{s_f(p)}$ is the elasticity of demand. Equation (8) shows that price dynamics cal, nonstop traffic. We do not model the potential for consumers to connect while flying between an origin-destination pair.

We observe bookings for consumers who purchased directly with the airline and on other booking channels, e.g., online travel agencies. We label these bookings *direct* and *indirect*, respectively. Because we observe all booking counts, we can construct the load factor for each flight over time. We do not know the exact itinerary involved for each booking, e.g., a round-trip versus a one-way booking. Therefore, we assume that the price paid for each nonstop booking corresponds to the lowest available nonstop, one-way fare for that flight.

Our pricing data come from a separate third-party data provider that gathers and disseminates fare information for the airline industry. The data frequency matches the booking information, i.e., we observe daily prices at the flight level. We observe fares even when there are no bookings. Several prices are tracked, including tickets of different qualities (cabins, fully refundable, etc.). We concentrate our analysis on the lowest available economy class ticket because travelers overwhelmingly purchase the lowest fare offered (Hortaçsu et al., 2021b). We do not model consumers choosing between cabins (economy vs. first class) nor the pricing decision for different versions of tickets.

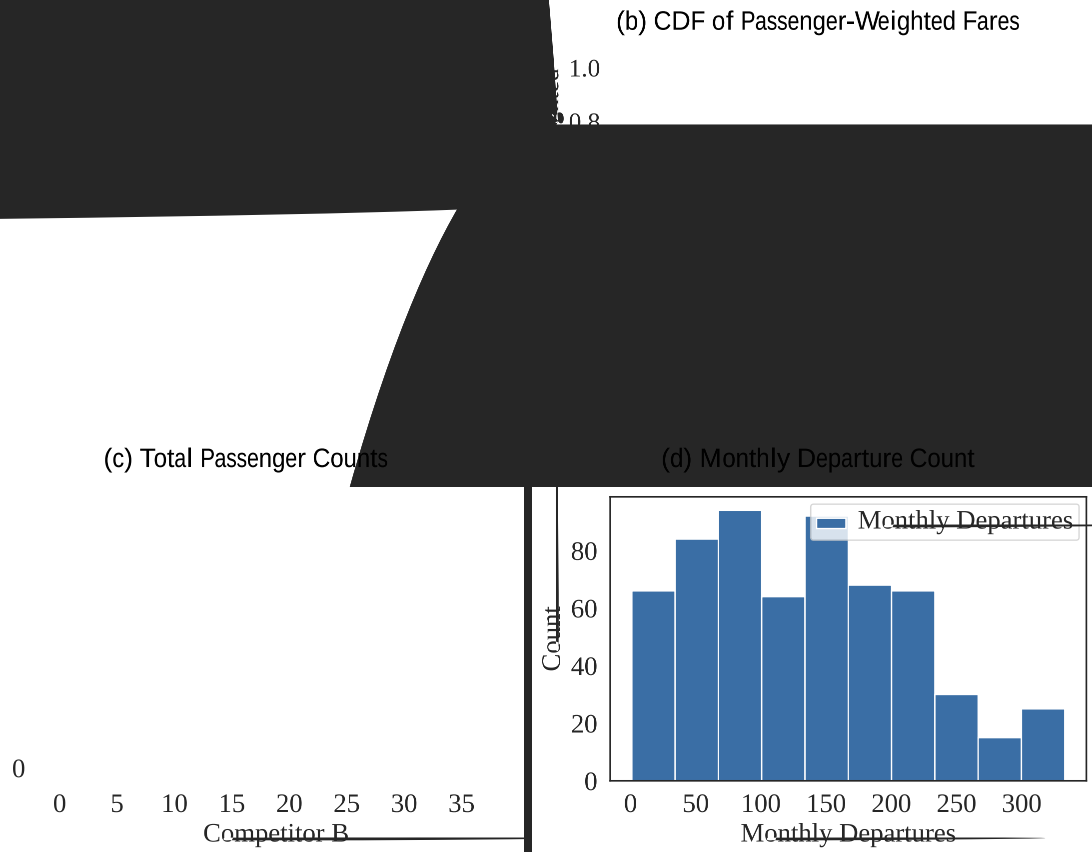
In order to gauge market sizes, we use clickstream search data provided to us by the air carrier. See Hortaçsu et al. (2021a) and Hortaçsu et al. (2021b) for more details. Observed searches understate true arrivals because some consumers may search and purchase through online travel agencies or directly with competitors. We extrapolate total arrivals by scaling up observed searches using hyperparameters that we describe below.

4.2 Route Selection

Our analysis concentrates on nonstop flight competition. We limit ourselves to routes where nonstop service is provided by exactly two airlines—by our data provider and one competitor. Our data contain more than one competitor airline, however, we will always refer to the competing airline as “the competitor.” We eliminate routes where the third-party data is

[redacted]rect bookings to the data provider but indirect
[redacted]iteria, we select routes in which most OD traf-
[redacted] allows us to avoid the additional complexity
[redacted]

[redacted]sis from the DB1B Data



Note: Panel (a) records the percenta[redacted]
(b) plots the cdf of prices for selec[redacted]
Panel (d) reports the number of ag[redacted]

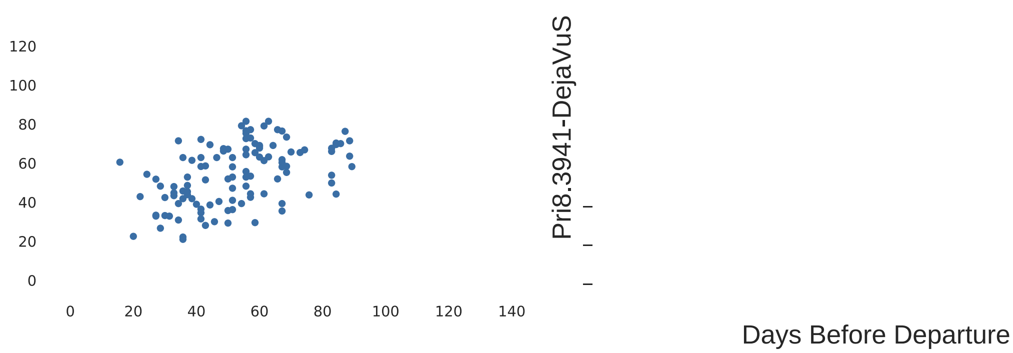
In Figure 3 we pro[redacted]
available DB1B data.[redacted]
on the booking and [redacted]
that is local versus [redacted]
all dual-carrier non[redacted]

[redacted]

both above and below the 45-degree line—one competitor does not consistently sell a larger fraction of capacity than the other carrier for all routes. We do observe some flights with substantial overselling. In our analysis, we restrict firms to selling at most their capacity. In the right panel we plot the average fare difference across competitors over time when exactly two flights are offered. Note that fares tend to be similar across competitors—the average difference is less than $10. However, the gradient of the prices differs. One competitor has relatively higher prices well in advance of departure and relatively lower prices close to departure. Note that for over 50% of the data, prices across firms are equal, that is, there is substantial price matching.

Figure 5: Load Factor and Price Differences across Carriers

(a) Load Factors (b) Mean Price Difference



Note: Panel (a) shows the average load factor (across all flights) at the route-departure date level for both competitors in blue. The orange squares report average route-level load factors. The diagonal line is the 45-degree line. Panel (b) shows the average and the 25th and 75 percentiles of the difference in prices for markets in which exactly two flights across firms are offered (one flight per airline).

5 Demand Model and Estimates

5.1 Empirical Specification

We model nonstop air travel demand using a nested logit demand model. Our model differs from recent empirical work on airlines that use a mixed-logit model to model “business” and “leisure” travelers (Lazarev, 2013; Williams, 2022; Aryal et al., 2021; Hortaçsu et al., 2021b). We use a flexible nested-logit model with time-varying as it better maps to our theoretical model and results in unique equilibrium price paths.[17]

Define a market as an origin-destination (r), departure date (d), and day before departure (t) combination. Each flight j, leaving on date d, is modeled across $t \in \{0, \ldots, T\}$. The first period of sale is $t = 0$, and the flight departs at T. We use a 90-day time horizon. With daily data, we model demand at the daily level. Arriving consumers choose flights from the choice set $J_{t,d,r}$ that maximize their individual utilities, or select the outside option, $j = 0$. There are two nests. The outside good belongs to its own nest, and all inside goods belong to the second nest.

We specify consumer arrivals to be

$$_{t,d,r} = \exp \quad ^{OD}_{r} + \quad ^{DD}_{d} + \quad ^{SD}_{t,d} + f\,(DFD)_t \quad ,$$

where denote fixed effects for the route, departure date, and search date; $f\,()$ is a polyno-

i chooses flight j if, and only if,

$$u_{i,j,t,d,r} \qquad u_{i,j^{0},d,t,r}$$

estimated arrival rates to account for unobserved searches. This follows from a property of the Poisson distribution and the assumption that consumers who search/purchase through alternative platforms (travel agents, other carriers' websites) have the same underlying preferences. We first calculate the fraction of direct bookings by day before departure and then scale up the estimated arrival rates using these these fractions. This adjusts arrivals for a single carrier. In our preferred specification, we then double these arrival rates to account for competitor indirect and direct searches, both of which are unobserved to us. We conduct robustness to this hyperparameter in Appendix D.

We summarize the demand estimates in Table 2. We estimate the nesting parameter to be 0.5 so that there is substantial substitution within inside goods. The price sensitivity parameters vary by nearly a factor of ten over time. We present a time series plot of $_t$ in Figure 6. Almost all of our controls are significant, with day of the week and week of the year having the most influence on market shares. The competitor FE is significantly less important in driving variation in shares. We estimate the average own-price elasticity to be -1.4.

Table 2: Demand Estimates Summary Table

Variable	Symbol	Estimate	Std. Error.	Range	% Sig.
Nesting Parameter		0.498	0.010		
Price Sens.				[-0.511 ,-0.074]	100.0
Competitor FE		0.071	0.003		
Day of Week FE				[-1.637 ,-0.961]	100.0
Departure Time FE				[-0.462 ,-0.050]	100.0
Route FE				[-0.177 ,0.226]	94.4
Week FE				[-0.953 ,0.699]	86.0
Sample Size	N		2,814,686		
Avg Elasticity	e^D		-1.438		

Note: Demand estimates for the 58 routes in our sample.

In Figure 6-(a), we plot average adjusted arrival rates as well as parts of the distribution (5%, 25%, 75%, 95%) across markets. We estimate just a few arrivals per market 90 days before departure that then increases to over 10 passengers per day close to departure. Recall that the average booking rate across ights is less than 2.0 (see Figure 4) so that market shares are low. An increase in interest in travel is a general ndings a-310( )-3a-31l310(-2

i) We consider only two products. Instead of investigating pricing in routes where we observe a single flight operating by each firm, we adjust the choice set, utilities, and capacities for all routes.

ii) We take the mean utilities across observed flights for each departure date and an input.

iii) We take the maximum observed capacity for each route-carrier-departure date. Although it may be natural to sum the capacities when restricting the choice set, we have found that large capacities presents a significant computational burden.

iv) We use the observed arrival process for each route-departure date. We do not adjust the estimated arrival processes as the inside good shares tend to be small. That is, because most consumers choose the outside good, we do not scale down arrival rates to account for smaller choice sets.

v) Finally, we handle flow (connecting traffic) bookings two ways. In our reported counterfactuals here, we model these bookings via Poisson processes that the firm does not internalize when pricing local demand. In the appendix we report counterfactuals where we subtract off all connecting bookings at the start of the game. This affects market outcomes because it reduces uncertainty for firms.

Benchmark Model

We approximate the continuous time model to solve for equilibrium prices for every departure date. We consider hourly decisions over 90 days. Both firms start with initial capacities K_f and $K_{f'}$. We solve via backward induction, which we outline here. In the last pricing period, $t = T$, both $_T(K) = 0$ and $_T(K) = 0$. Therefore, both firms solve static revenue maximize problems. We set the first-order conditions corresponding to the best response functions equal to zero and solve for the fixed point. We denote the fixed-point price vector by $p_T = p(\ _T, \ _T)$ where $_T = 0$. Let us denote the stage-game payoff of firm f in period t

given price vector p and by $_{f,t}(p,)$. Then, using the differential equation, we can write $\dot{}_{f,T}(K) = {}_{T}\ {}_{f,T}(p_T(K), 0)$, which allows us to calculate ${}_{f,T}(K) = {}_{f,T}(K)\ \dot{}_{f,T}(K)$ and $!^{f}_{f0,T}(K) = {}_{f,T}(K)\ {}_{f,T}(K\ e_{f0})$. Given the updated own and scarcity effect parameters we again solve for equilibrium prices, $p_T = p_T(\ {}_T,\ {}_T)$.[18] We continue the induction backwards in time to $t = 0$.

Due to the large number of state variables, we store $_t$ and p_t every 24 hours (at the start of a day) in order to use them in counterfactual simulations. We then appeal to modeling demand via multinomial distributions after drawing arrivals from a Poisson distributions in lieu of studying each consumer's individual choice after drawing arrivals from a Bernoulli distributions as in the theoretical model.

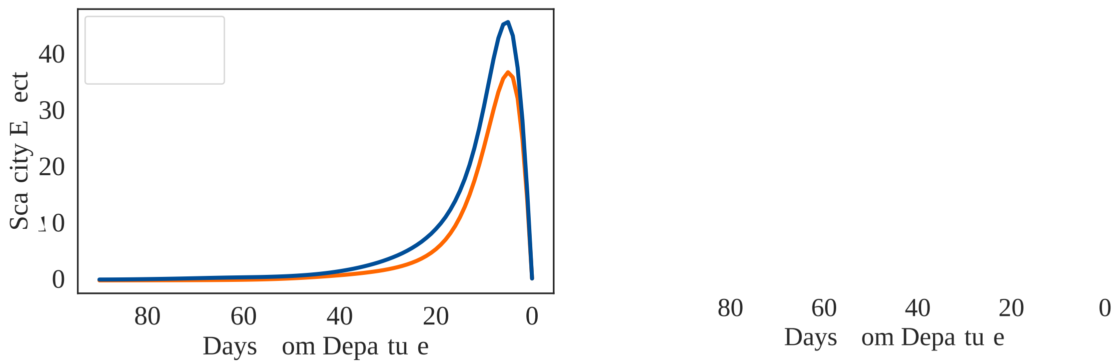
6.0.1 Pricing with Heuristics

We compare the benchmark model to two pricing heuristics where firms do not internalize the scarcity of their competitor and firms also do not explicitly account for the fact that their competitor is a strategic agent solving a dynamic pricing problem. In both heuristics, we consider discrete prices as they are used in actual airline pricing practices. Applied theory 0p

Figure 7: Benchmark Model Opportunity Costs

(a) Own Omega

(b) Competitor Omega

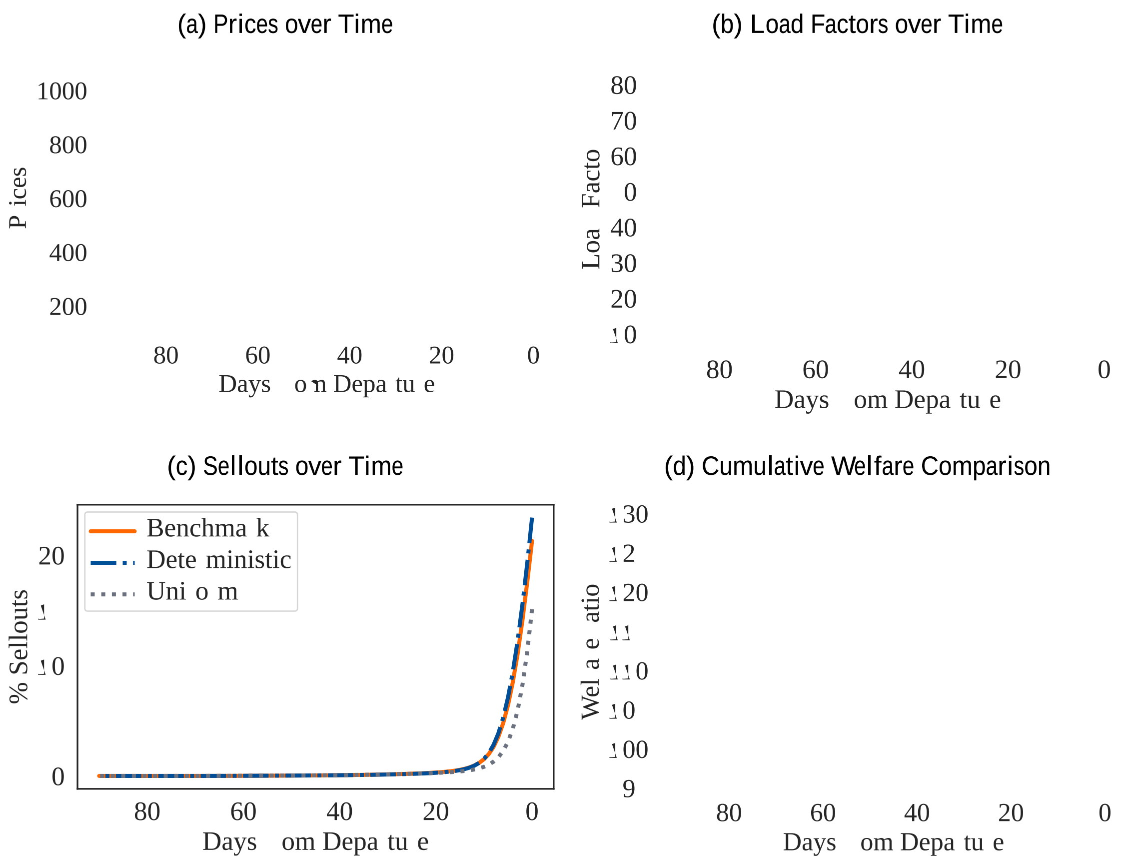


Note: Panel (a) reports the own-firm scarcity effect over time for both firms. Panel (b) reports the cross-firm competitor scarcity effect over time for both firms.

Table 3: Counterfactual Results

	Price	Firm 1 Rev.	Firm 2 Rev.	CS	Welfare	Q	LF	Sellouts
Benchmark	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Deterministic	98.3	96.8	97.6	108.4	103.9	103.2	101.2	109.9

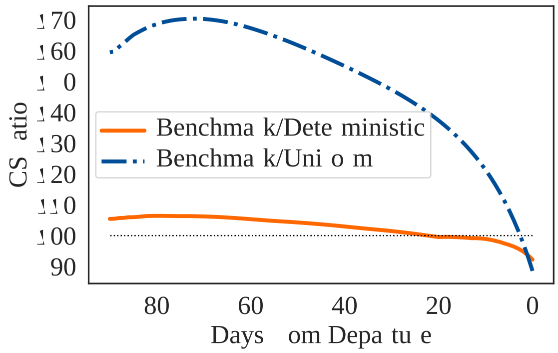
Figure 8: Counterfactual Summary Plots



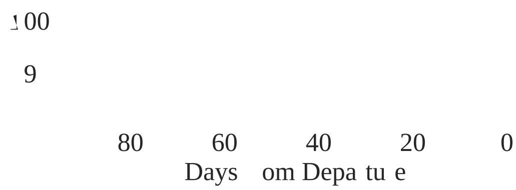
Note: Panel (a) shows the average price over time for the benchmark, deterministic and uniform models. Panel (b) shows the average load factors over time for the same three models. Panel (c) shows the average sellouts over time for the same three models. Panel (d) shows the ratio of average cumulative welfare for the benchmark model with respect to the deterministic one, and for the benchmark model with respect to the uniform one.

Figure 9: Cumulative Surplus Differences Across Counterfatuals

(a) Cumulative CS Difference



(b) Cumulative Revenue Difference



Note: Panel (a) reports the own-firm scarcity effect over time for both firms. Panel (b) reports the cross-firm competitor scarcity effect over time for both firms.

Figure 10: Heuristic Counterfactuals Prices and Load Factors

(a) Prices over Time

(b) Load Factors over Time

Note: Panel (a) shows the average prices over time for the two heuristic models. Panel (b) shows the average load factors over time for the same two models.

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A General Model with Many Firms and Many Products

In Appendix A, we formulate the generalized results stated in Section 3 for the duopoly case. We directly prove those general statements in Appendix B.

A.1 Model Setup

We consider a market with $F \geq 1$ firms and $J \geq F$ products, denoting the set of firms by $\mathcal{F} := \{1, \ldots, F\}$ and the set of products by $\mathcal{J} := \{1, \ldots, J\}$. Each firm f sells products in $\mathcal{J}_f$, where $(\mathcal{J}_f)_{f \in \mathcal{F}}$ is a partition of $\mathcal{J}$; that is, $\mathcal{J} = \bigcup_{f \in \mathcal{F}} \mathcal{J}_f$ and $\mathcal{J}_f \cap \mathcal{J}_{f'} = \emptyset$ for $f \neq f'$. Thus, no product is sold by more than one firm. Each firm f is equipped with an initial inventory of their products $j \in \mathcal{J}_f$, denoted by $K_{j,0} \in \mathbb{N}$. We assume that the demand system for the products in $\mathcal{J}$ is as introduced in Section 2.1, and satisfies Assumptions 1 and 2.

The dynamic pricing game is the canonical generalization of the duopoly game introduced in Section 2.3. In every period t, each firm f simultaneously sets prices $p_{j,t}$ for its products $j \in \mathcal{J}_f$, and then a consumer arrives with probability with boundary conditions (i) $_{f,T+}(K;\ )$ 0 for all K, (ii) $_{f,t}(K;\ )$ 0 if $K_j = 0$ for all $j \in J_f$ and (iii) $_{f,t}(K;\ ) = 1$ if $K_j < 0$ for a $j \in J_f$, (iv) $_{f,t}(K \quad e_j;\ ) = \ _{f,t}(K;\ )$ if $K_j = 0$ for a $j \notin J_f$ K_j 0 for all $j \in J_f$.. Then, we denote the

Then, we can generalize Assumption 3 and Lemma 2 as follows.

General Assumption 3. *The following two conditions hold,*

i) $D_{p_f} g_f(p) \quad 1 \neq 0$ *for all* p *and* f *;*

ii) $\det \; D_p \, g(p) \quad I \quad \neq 0$ *for all* p*, where* $g(p) := (g_f(p))_{f \in F}$ *.*

General Lemma 2. *Let Assumptions 1, 2, and General Assumption 3 hold. Then, the stage game admits a unique equilibrium.*

A.2.2 Continuity of Equilibrium Prices in Scarcity Effect Matrix

General Lemma 3. *If the equilibrium of the stage game is unique for a compact set* O *of costs , then there exists an equilibrium price vector* p (,) *that is continuous in on* O *and on .*

A.2.3 Characterization of Continuous-time Limit

General Proposition 2 (Continuous-time limit Limit)**.** *Let Assumptions 1, 2, and General Assumption 3 hold for* $=0$*. For every* K*, there exists a* $T_0(K) > 0$*, non-increasing in* K*, so that for any* T $T_0(K)$ *there exists a unique equilibrium of the dynamic pricing game for sufficiently small . Then, there exists a unique subgame-perfect equilibrium. The value function* $_{f,t}(K; \;)$ *converges to a limit* $_{f,t}(K)$ *that solves the differential equation*

$$\dot{}_{f,t}(K) = \quad_t \sum_{j \in J_f} s_j(p(\;_t(K); \;))\; p_j(\;_t(K); \;_t) \quad (\;^f_t(K) \quad ^f_t(K \quad e_j)) \quad \sum_{j' \notin J_f} s_{j'}(p(\;_t(K); \;)) \quad ^f_t(K) \quad ^f_t(K \quad e_{j'})$$

K

[15Tf 123.986 0 Td [(˝)]TJ/F144 7.4919 Tf 8.942 6.236 Td [7 8. Tf3

with boundary conditions $^f_t(K) = 0$ *if* K 0

Lemma 4. *For a logit demand system as defined in Equation 3, holding everything else fixed, there exists a* $\bar{}$ *and a* $\bar{}$ > 0 *so that for all* $> \bar{}$ *and* $< \bar{}$*, the cost matrix* $_t(K)$ *satisfies Assumption 4 for all* t 2 $[0, T]$ *and* K K_0*.*

A.2.4 Additional Theoretical Results on Dynamic Price Competition

Capacity Distribution and Prices Assume that $_t$ and $s_{f,t}$ is independent of time, i.e.,

. *Further, there exist functions* $d_j(p_{-f};\ ,\ )$ *so that the equilibrium prices of the stage game coincide with the equilibrium prices of a game with a set* J *of players who each simultaneously choose a price* p_j *maximizing*

$$s_j(p)(p_j - c_j(p_{-j};\ ,\ )) + d_j(p_{-j};\ ,\ )$$

with a cost function

$$c_j(p_{-j};\ ,\ ) := \omega_j^f \sum_{j' \in J_f \setminus \{j\}} \tilde{s}_{j,j'}\, p_{j'}(p_j,\ ^f \ldots$$

Td1(
s[Td [( )]TJ0(+)] T1/F144 11.2379 Tf 19.534 7d [9 Td [(X)

Tf-294-149.478 3.578(
d [(s)]TJ/F144 11.2379 Tf 0.219 -0.191 Td [(s)]TJ/F144 7.4919 Tf 5.308 -2.259 Td [8i

Lemma 5. *Consider a continuous price function* $(\ , \ ) 7! \ p\ (\ , \ ) = (p_f (\ , \ ))_f$ *on a compact set* O, *and a bounded and continuous function* $A : R^F \quad R^{J\ F}$ *. Let* $\ _{f,t}(K; \ )$, $f\ 2F$, *be a solution to the difference equations*

$$\frac{\ _{f,t+}\ (K; \ ) \qquad _{f,t}(K; \ )}{\ } \ _f = \ _t A\ p \left((K; \ )), \ _t\ , \ (K; \ ) \right)$$

where $(K; \ ) = (\ !^{\ f}_{j,t}(K; \ ))_{f,j}$, $!^{\ f}_{j,t}(K; \ ) := \ _{f,t+}\ (K; \ ) \quad _{f,t+}\ (K \quad e_j; \ )$, *with boundary conditions (i)* $\ _{f,T+}\ (K; \ ) = 0$, *(ii)* $\ _{f,t}(K; \ ) = 0$ *if* $K_j = 0$ *for all* $j\ 2\ J_f$, *(iii)* $\ _{f,t}(K; \ ) = 1$ *if* $K_j < 0$ *for a* $j\ 2\ J_f$, *and (iv)* $\ _{f,t}(K \quad e_j; \ ) = \ _{f,t}(K; \ )$ *if* $K_j = 0$ *for a* $j\ 62\ J_f$, $K_{j^0} \quad 0$ *for all* $j^0 2\ J_f$*. Then,* $(\ ^f_t(K; \ ))_f$ *converges to a limit* $(\ _{f,t}(K))_f$ *that satisfies*

$$\dot{\ }_{f,t}(K)\ _f = \ _t A\ p \left((K), \ _t\ , \ (K) \right),$$

where $(K) = (\ !^{\ f}_{j,t}(K))_{f,j}$, $!^{\ f}_{j,t}(K) := \ _{f,t}(K; \ ) \quad _{f,t+}\ (K \quad e_j)$, *with boundary conditions (i)* $\ _{f,T}(K) = 0$, *(ii)* $\ _{f,t}(K) = 0$ *if* $K_j = 0$ *for all* $j\ 2\ J_f$, *(iii)* $\ _{f,t}(K;) = 1$ *if* $K_j < 0$ *for a* $j\ 2\ J_f$, *and (iv)* $\ _{f,t}(K \quad e_j) = \ _{f,t}(K)$ *if* $K_j = 0$ *for a* $j\ 62\ J_f$, $K_{j^0} \quad 0$ *for all* $j^0 2\ J_f$.

Proof. Since A is bounded, the difference equations show that $(\ _f(K; \ ))_{f\ 2F\ ,K\ \ K_0}$ is equicontinuous and equibounded in t as $\ ! \ 0$. Hence, by the Arzela-Ascoli Theorem, there exist limit points $(\ _f(K))_{f\ 2F\ ,K\ \ K_0}$. We claim that

$$\ _{f,t}(K)\ _f = \int_t^T \ _u A\ p \left(\ _u(K), \ _u\ , \ _u(K) \right) du. \tag{12}$$

To this end, we note that if we let $d u e$ to be the largest number that is divisible by and smaller or equal than u

$$\ _{f,t}(K; \ )\ _f = \int_t^T \ _{d u e}\ A\ p \left(\ _{d u e}(K; \ ), \ _{d u e}\ , \ _{d u e}(K; \ ) \right) du. \tag{13}$$

We take the limit ! 0 on both sides. The left-hand side of (13) converges to the left-hand side of (12). On the right-hand side, $_{due}$(K;) converges to $_{u}$(K). Hence, by continuity of p and A the integrand in (13) converges to the integrand in (12). The dominated convergence theorem finishes the proof. „

B.1.2 Continuity of stage game prices

Lemma 6. *Let* P R^J *be compact and convex and* O *a path-connected set of* (,). *Further, let* g : (q; ,) 7! p *be defined as a function* P O ! P *, where* g *is continuously differentiable in* q *and continuous in* *and* *. If* det(D_qg(q; ,) I) 6= 0 *for all* (q; ,) 2 P *, then there is a unique* p (,) *satisfying* g(p ( ); ,) = p (,) *and it depends continuously on* *and* *.*

Proof. The existence and uniqueness of p (,) follows directly from Lemma 2 (Kellogg (1976)) in Konovalov and Sándor (2010). To show continuity, we consider a sequence ($_n$, $_n$)$_{n\ 1}$ converging to some ($_1$, $_1$). Thanks to path-connectedness of O there exists a continuous path r : [0,1] ! O and a sequence a_n " 1 such that r(a_n) = ($_n$, $_n$) and r(1) = ($_1$, $_1$). By Browder's Theorem (Theorem 1.1 in Solan and Solan (2021)), the set f(p (r(a)); a) : a 2 [0,1]g P [0,1] is connected. By the main theorem of connectedness, each set f(p_j (r(a)); a) : a 2 [0,1]g R [0,1] is connected, for all j. By Burgess (1990), the function a 7! p_j (r(a)) is continuous, so p_j ($_n$, $_n$) = p_j (r(a_n)) ! p_j (r(1)) = p_j ($_1$, $_1$).
„

B.2 Proofs of Single Firm Model

B.2.1 Proof of Lemma 1

The profit-maximizing prices of the stage game p_M(!) are implicitly given by (4).

$$g(q;!\ ,\ ) := !\ \ (D_p s_t$$

is continuously differentiable in q, and by Assumption 1, and any fixed point must satisfy q ! and q ! +1 by Assumption **??** iii). Hence, the convergence to 4 follows by Lemma 5.

B.2.2 Proof of Proposition 1

Proof. i) To see that $_{M,t}$ is decreasing in t, note that in (4), p_j can always be chosen so that objective function in the maximum is positive. Hence, $\dot{}^{M}_{t}(K) < 0$.

Next, we show that $^M_t(K) > {}^M_t(K \quad e_j)$ for all j by induction in $\sum_j K_j$.

Induction start: It is immediate that $^M_t(e_j) \quad {}^M_t(0) = 0$ for all j and t T.

Induction hypothesis: Assume that $^M_t(K) > {}^M_t(K \quad e_j)$ for all K such that $\sum_j K_j = \bar{K}$.

Induction step: Now, consider a capacity vector K with $\sum_j K_j = \bar{K} + 1$. By sub-optimality of the prices $p^M(! \ {}^M_t(K \quad e_{\tilde{j}}))$ given capacity vector K, we have

$$^M_t \; K \quad \int_t^T e^{\int_t^u \sum_{j''} s_{j'',u}(p_u} \bullet \sum_j s_{j,z} \; p^M(! \ {}^M_z(K \quad e_{\tilde{j}})) \; p^M_{j,z}(! \ {}^M_z(K \quad e_{\tilde{j}})) + {}^M_z(K \quad e_j)$$

$H(0;\ ) = 0$ by Assumption **??** iii). Since H is concave, it admits the representation

$$H(x;\ ) = \inf_s (s\ x \quad H\ (s;\ ))$$

where the concave $H\ (s;\ ) = \inf_x (x\ s \quad H(x;\ ))$ is the concave conjugate of H, with $H\ (0;\ ) = 0$. Moreover,

$$\dot{\ }^M_t(K) = \ _t H(r \ \ _t(K);\ \ _t)$$

where $r\ ^M_t(K) = \ ^M_t(K) \quad ^M_t(K \quad e_j)\ _j$. Thus, $^M_t(K)$ is the value function for the optimal control problem

$$^M_t(K) = \sup_{s2A} E \left[\int_t^T \ _u H\ (s_u;\ _u) du \ \ X^s_t = K \right] =: \sup_s \tilde{J}_t(K, s)$$

s

where X^a_t is the process which jumps by e_j at rate $\tilde{s}_j$, and $s_t\ K$, $s\ 2\ A$ are processes adapted

[(])]TJ/F34 4.491 p50 Td [(s)(.308j-24918s.3)]TJ]TJ/F8 8 11.9552 Tf 8.789 4.718 Td [(68096e Tf 5.8

with respect to X^s, with the property $s_{j,t} = 0$ if $X^s_{j,t} = 0$ (Theorem 8.1 in Fleming and Soner (2006)). Let s_K be the optimal control in the previous equation and $s_{K\ 2}$ be the optimal control when K is replaced by $K \quad 2e_j$. Then, note that since $s_K, s_{K\ 2e_j}\ 2\ A$, $\frac{s_K + s_{K\ 2e_j}}{2}\ 2\ A$ because the process $X_s^{\frac{s_K + s_{K\ 2e_f}}{2}}$ s can be chosen as $\frac{X_s^{s_K} + X_s^{s_{K\ 2e_f}}}{2}$ s

H

iii) To show that $!^{M}_{j,t}(K_t)$ is a submartingale, we show that for any capacity vector K,

$$\lim_{! \; 0} \frac{E_0 \left[!^{M}_{j,t+}(K_{t+}) \quad !^{M}_{j,t}(K_t) \, K_t = \bar{K} \right]}{} \quad 0.$$

To this end, first, note that K_t is right-continuous in t. Further, for K with $K_j = 0$, we set $!^{M}_{j,t}(K) = 1$ for all t. Thus, we are setting the opportunity cost of selling a unit if no capacity is left to infinity, which is equivalent to the constraint of not being able to sell units that are not available.

Then, we have for $\bar{K}$ with $\bar{K}_j = 1$ that

$$\lim_{! \; 0} \frac{E_0 \left[!^{M}_{j,t+}(K_{t+}) \quad !^{M}_{j,t}(K_t) j K_t = \bar{K} \right]}{} > 0.$$

Next consider $\bar{K}$ with $\bar{K}_j \quad 0$. Then, we have that

$$\lim_{! \; 0} \frac{E_0 \left[!^{M}_{j,t+}(K_{t+}) \quad !^{M}_{j,t}(K_t) j K_t = \bar{K} \right]}{} =$$

$\lim_{! \; 0}$ E0 Td [((lim))]TJ/F145 7.4919 Tf 10.32 -7.0669 Td [(set-7.084 Td [(t)]TJ/F140 7.4919 Tf 6.091 0 Td [(!)]TJ/F141 7.4919 Tf 7.357 0 Td [(0)]TJ/F15 t. Further, for K with

lim
set-7.084 Td [( )]TJ/F140 7.4919 Tf 6.091 0 Td [( )]TJ/F141 7.4919 Tf 7.357 0 Td [(0)]TJ/F154 11.23794 in tt. Further(j K€

Hence, $\lim_{! \, 0} \frac{E_0 \; ! ^{M}_{j,t+} \; (K_{t+} \;) \;\; ! ^{M}_{j,t} (K_t) | _t = \bar{K}]}{}$ is equal to

$$\sum_t \sum_{j^0} s_{j^0,t} \; p^M_t (\bar{K}) \;\; p^M_{j^0,t} (\bar{K}) \qquad ! ^M_{j^0,t} (\bar{K} \quad e_j) \qquad s_{j^0,t} \; p^M_t (\bar{K} \quad e_j) \;\; p^M_{j^0,t} (\bar{K} \quad e_j) \qquad ! ^M_{j^0,t} (\bar{K} \quad e_j)$$

Then, note that by definition of $p^M_t (\bar{K} \quad e_j)$,

$$\sum_{j^0} s_{j^0,t} \;\; p^M_t (\bar{K}) \;\; p^M \qquad ! \qquad e_j) \;\; j)$$

M43 10.!f) 5.098 0 Td [( )]TJ/F1173 0 Td [(t)]TJ/F164 11.2379 Tf 4.461 12.3F164 11.2379 Tf -2J/F144 7.4919 Tf 8.035 5.034 T

$s_{j^0,t}$

0

$$0 \geq \underbrace{\frac{\partial s_j}{\partial p_j}}_{<0}(p_j - \omega_j^f) + \sum_{k \in J_f \setminus \{j\}} \underbrace{\frac{\partial s_k}{\partial p_j}}_{>0}(p_k - \omega_k^f)$$

$$- \sum_{k \notin J_f} \frac{\partial s_k}{\partial p_j}(q^{-f}, p^f)\, \omega_k^f + s_j(q^{-f}, p^f)$$

$$\geq -\frac{\partial s_j}{\partial p_j}(q^{-f}, p^f) \left(\frac{1}{J-1} \sum_{k \in J_f \setminus \{j\}} (p_k - \omega_k^f)_+ - 2(p_j - \omega_j^f) + \underbrace{\frac{s_j(q^{-f}, p^f)}{-\frac{\partial s_j}{\partial p_j}(q^{-f}, p^f)}}_{\text{by Assumption ??-??}} \right.$$

$$\left. + \sum_{k \notin J_f} \frac{|\omega_k^f|}{J-1} \right).$$

This is equivalent to

$$\forall j \in J_f : \quad 2(p_j - \omega_j^f) + \frac{s_j(q^{-f}, p^f)}{-\frac{\partial s_j}{\partial p_j}(q^{-f}, p^f)} \quad \frac{1}{J-1} \sum_{k \in J_f \setminus \{j\}} (p_k - \omega_k^f)_+ + \sum_{k \notin J_f} \frac{|\omega_k^f|}{J-1}$$

$$\Rightarrow \quad \forall j \in J_f : \quad 2(p_j - \omega_j^f) - \frac{1}{J-1} \sum_{k \in J_f \setminus \{j\}} (p_k - \omega_k^f(p_k \quad {}^f$$

This is equivalent to

$$8\ j\ 2\ J_{f} :\quad 2(p_{j}\quad !_{j}^{f}) + s_{j}(q^{f}, p^{f})$$

We are interested in the limit as $t\ !\ \ T$. First, $\lim_{t\,!\,\,T}\ \ {}_t = 0$. Furthermore, we can write

$$
\begin{aligned}
!^{f}_{j,t}(K) &= \dot{}^{f}_{t}(K) \quad \dot{}^{f}_{t}(K \quad e_j) \\
&= \quad s^{f}(p_t(K))p^{f\,\prime}_{t}(K) \quad s(p_t(K))!\ {}^{f}_{t}(K) \quad (s^{f}(p_t(K \quad e_j))p^{f\,\prime}_{t}(K \quad e_j) \quad s(p_t(K \quad e_j))!\ {}^{f}_{t}(K \quad e_j))
\end{aligned}
$$

Thus, as $t\ !\ \ T$, $!^{f}_{j,t}(K) = 0$ if $K_j > 1$. If $j\ 2\ J_f$ and $K_j = 1$, then $!^{f}_{j,t}(K) < 0$. If $j\ 62\ J^f$ and $K_j = 1$, then by the competition effect $!^{f}_{j,t}(K) > 0$.

This implies that $\dot{p}_{j,T}(K) < 0$ if $K_j = 1$ and $\dot{p}_{j,T}(K) = 0$ otherwise.

Induction assumption: If $K_j > n \quad 1$ for all are given by

$$p_j - \omega_j^f - \sum_{j'\in J_f\setminus\{j\}} \frac{\frac{\partial s_{j'}(p)}{\partial p_j}}{\frac{\partial s_j(p)}{\partial p_j}}(p_{j'} - \omega_j^f) + \sum_{j'\notin J_f} \frac{\frac{s_{j'}(p)}{\partial p_j}}{\frac{\partial s_j(p)}{\partial p_j}}\omega_{j'}^f = -\frac{s_j(p)}{\frac{\partial s_j(p)}{\partial p_j}}.$$

Further, the observation that $\frac{\partial s_j(p)}{\partial p_j} = -\sum_{j'\in J^0\setminus\{j\}} \frac{\partial s_{j'}(p)}{\partial p_j}$, and by Assumption **??** (Independence of Irrelevant alternatives) it follows that

$$\begin{aligned}
c(p_{-j};\omega) &:= \omega_j^f - \sum_{j'\in J_f\setminus\{j\}} \frac{\frac{\partial s_{j'}(p)}{\partial p_j}}{\frac{\partial s_j(p)}{\partial p_j}}(p_{j'} - \omega_j^f) + \sum_{j'\notin J_f} \frac{\frac{s_{j'}(p)}{\partial p_j}}{\frac{\partial s_j(p)}{\partial p_j}}\omega_{j'}^f \\
&= \omega_j^f - \sum_{j'\in J_f\setminus\{j\}} \frac{\frac{\partial s_{j'}(p)}{\partial p_j}}{-\sum_{j''\in J^0\setminus\{j\}}\frac{\partial s_{j''}(p)}{\partial p_j}}(p_{j'} - \omega_j^f) + \sum_{j'\notin J_f} \frac{\frac{s_{j'}(p)}{\partial p_j}}{-\sum_{j''\in J^0\setminus\{j\}}\frac{\partial s_{j''}(p)}{\partial p_j}}\omega_{j'}^f \\
&= \omega_j^f - \sum_{j'\in J_f\setminus\{j\}} \tilde{s}_{-j,j'}(p_{-j})(p_{j'} - \omega_j^f) + \sum_{j'\notin J_f} \tilde{s}_{-j,j'}(p_{-j})\,\omega_{j'}^f.
\end{aligned}$$

Thus, the first-order conditions of the stage game are equivalent to the first order conditions of a game with J players where each player j's payoff is given by

$$s_j(p)(p_j - c(p_{-j};\omega)) + d(p_{-j};\omega).$$

Existence Assume $s_j(p;\ ) > 0$ satisfies Assumptions **??** and **??**. Then, we define the best-response function of "player" j in the game defined in Lemma 4 by

$$R: q \mapsto \arg\max_{p_j} s_j(q)(p_j - c_j(q_{-j};\omega,\ )) = \ _0\ \text{spolis907 Td [(;)]TJ/F152 g}$$

j

First, we show that R is well-defined as a function R^J 7! [1 , 1]^J (rather than a correspondence). To this end, note that player j 's profit is increasing in p_j if and only if

$$p_j \quad c_j(q \; _j;! \; ,$$

C Simple Logit and Nested Logit Calculations

C.1 Simple Logit Demand

Consider a logit logit demand system as specified in Equation 3: $s_j(p) = \frac{e^{\delta_j - \alpha p_j}}{1+\sum_{j \in J} e^{\delta_j - \alpha p_j}}$. Throughout this section we omit the arguments of the demand functions when there is no ambiguity. First, note that

$$\frac{\partial s_j}{\partial p_j} = -\alpha s_j(1-s_j) \qquad \frac{\partial s_j}{\partial p_{j'}} = \alpha s_j s_{j'}.$$

First, we show that Assumption 1 is satisfied.

i) $\lim_{p_j \to \infty, \forall j \notin A}$

First, we show that Assumption 2 is satisfied. To this end, note that

$$(D_p s(p;\theta))^{-1} = -\frac{1}{\alpha}\begin{pmatrix} s_1(1-s_1) & -s_1 s_2 & \dots & -s_1 s_J \\ -s_2 s_1 & \ddots & & -s_2 s_J \\ \vdots & & \ddots & \vdots \\ -s_J s_1 & \dots & -s_J s_{J-1} & s_J(1-s_J) \end{pmatrix}^{-1} = -\frac{1}{\alpha s_0}\begin{pmatrix} 1+\frac{s_0}{s_1} & 1 & \dots & 1 \\ \vdots & & \ddots & \vdots \\ 1 & \dots & 1 & 1+\frac{s_0}{s_J} \end{pmatrix}.$$

Hence,

$$\Delta = (D_p s(p;\theta))^{-1} s(p;\theta) = -\frac{1}{\alpha s_0}\begin{pmatrix} 1+\frac{s_0}{s_1} & 1 & \dots & 1 \\ \vdots & & \ddots & \vdots \\ 1 & \dots & 1 & 1+\frac{s_0}{s_J} \end{pmatrix}\begin{pmatrix} s_1 \\ \vdots \\ s_J \end{pmatrix} = -\frac{1}{\alpha s_0}\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

and since $\frac{\partial}{\partial p_j}\frac{1}{s_0} = -\alpha\frac{s_j}{s_0}$,

$$\det\left(D_p \Delta - I\right) = \det\begin{pmatrix} \frac{s_1}{s_0}-1 & \dots & \frac{s_J}{s_0} \\ \vdots & \ddots & \vdots \\ \frac{s_1}{s_0} & \dots & \frac{s_J}{s_0}-1 \end{pmatrix} = (-1)^J \frac{1}{s_0}$$

$$(D_{p^f} s^f(p;\sigma))^{-1} s^f(p;\sigma) = -\frac{1}{s_0^f}\begin{pmatrix}1\\ \vdots \\ 1\end{pmatrix}$$

Finally,

$$g^f(p) = \begin{pmatrix} \frac{\sum_{j\in J\setminus J^f} s_j}{1-\sum_{j\in J^f} s_j}\,\omega_j^f - \omega_1^f - \frac{1}{1-\sum_{j\in J^f} s_j} \\ \vdots \\ \frac{\sum_{j\in J\setminus J^f} s_j}{1-\sum_{j\in J^f} s_j}\,\omega_j^f - \omega_{J_f}^f - \frac{1}{1-\sum_{j\in J^f} s_j} \end{pmatrix}$$

and

$$D_p g^f(p) = \begin{pmatrix} \begin{pmatrix} \frac{s_k}{1-\sum_{j\in J^f} s_j} \\ \vdots \\ \frac{s_k}{1-\sum_{j\in J^f} s_j} \end{pmatrix}_{k\in J^f}, \begin{pmatrix} -\frac{(1-\sum_{j\in J^f} s_j)(1-\sum_{j\notin J^f} s_j)+(\sum_{j\in J^f} s_j)(\sum_{j\notin J^f} s_j)}{(1-\sum_{j\in J^f} s_j)^2}\,\omega_j^f + s_k\frac{\sum_{j\in J^f}}{(1-\sum_{j\in J^f} s_j)^2} \\ \vdots \\ -\frac{(1-\sum_{j\in J^f} s_j)(1-\sum_{j\notin J^f} s_j)+(\sum_{j\in J^f} s_j)(\sum_{j\notin J^f} s_j)}{(1-\sum_{j\in J^f} s_j)^2}\,\omega_j^f + s_k\frac{\sum_{j\in J^f}}{(1-\sum_{j\in J^f} s_j)^2} \end{pmatrix}_{k\notin J^f} \end{pmatrix}$$

For large σ the term in front of ω_j^f vanishes
-6 [(9 QQA)]TJ/F86 i89 Tf -3.321 -13.895 Td [(j)]TJC

Further, $s(p;\ )^{ü}! = \sum_j s_j\,!_j^f$ and

$$D_{p^f}(s(p;\ )^{ü}!\ ) = -\begin{pmatrix} s_1 \sum_{j2J} s_j\,!_j^f & !_1^f \\ \vdots & \\ s_{J_f} \sum_{j2J} s_j\,!_j^f & !_{J_f}^f \end{pmatrix}$$

Hence,

$$(D_{p^f} s^f(p;\ ))^{1} D_{p^f}(s(p;\ )^{ü}!\ ) = \frac{1}{s_0^f}\begin{pmatrix} \sum_{j2J_f} s_j + s_0^f & \sum_{j2J} s_j\,!_j^f & \sum_{j2J_f} s_j\,!_j^f & s_0^f\,!_1^f \\ & \vdots & & \\ \sum_{j2J_f} s_j + s_0^f & \sum_{j2J} s_j\,!_j^f & \sum_{j2J_f} s_j\,!_j^f & s_0^f\,!_{J_f}^f \end{pmatrix}$$

$$= \frac{1}{s_0^f}\begin{pmatrix} \sum_{j2JnJ_f} s_j\,!_j^f & s_0^f\,!_1^f \\ \vdots & \\ \sum_{j2JnJ_f} s_j\,!_j^f & s_0^f\,!_{J_f}^f \end{pmatrix}$$

Further, since

$$(D_{p^f} s^f(p;\ ))^{1} s^f(p;\ ) = = \frac{}{s_0^f}\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

Finally,

$$g^f(p) = \begin{pmatrix} \frac{\sum_{j2JnJ_f} s_j}{1 \sum_{j2J_f} s_j}\,!_j^f & !_1^f + \frac{1}{1 \sum_{j2J_f} s_j} - \\ \vdots & \\ \frac{\sum_{j2JnJ_f} s_j}{1 \sum_{j2J_f} s_j}\,!_j^f & !_{J_f}^f + \frac{1}{1 \sum_{j2J_f} s_j} - \end{pmatrix}$$

and

$$D_p g^f(p) = \left(\begin{pmatrix} \frac{s_k}{1-\sum_{j \in J_f} s_j} \\ \vdots \\ \frac{s_k}{1-\sum_{j \in J_f} s_j} \end{pmatrix}_{k \in J_f}, \begin{pmatrix} -\frac{(1-\sum_{j \in J_f} s_j)(1-\sum_{j \notin J_f} s_j)+(\sum_{j \in J_f} s_j)(\sum_{j \notin J_f} s_j)}{(1-\sum_{j \in J_f} s_j)^2}\, \omega_j^f + s_k \frac{\sum_{j \in J_f} s_j}{(1-\sum_{j \in J_f} s_j)^2} \\ \vdots \\ -\frac{(1-\sum_{j \in J_f} s_j)(1-\sum_{j \notin J_f} s_j)+(\sum_{j \in J_f} s_j)(\sum_{j \notin J_f} s_j)}{(1-\sum_{j \in J_f} s_j)^2}\, \omega_j^f + s_k \frac{\sum_{j \in J_f} s_j}{(1-\sum_{j \in J_f} s_j)^2} \end{pmatrix}_{k \notin J_f} \right)$$

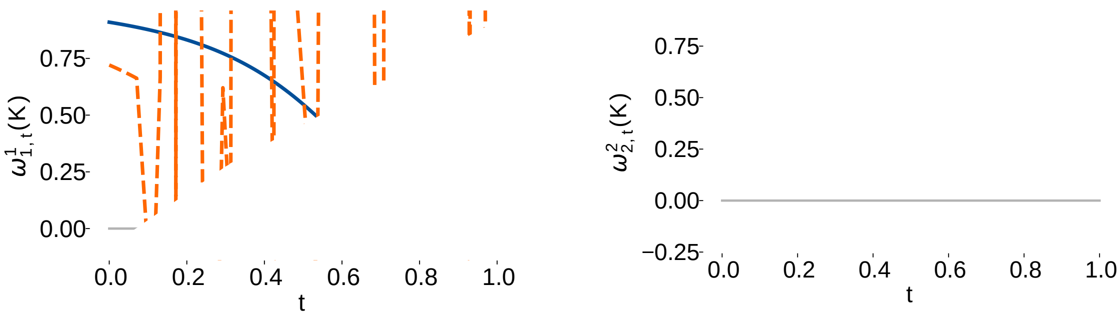
For large the term in front of ω_j^f vanishes relative to the probability.

D Additional Empirical Results

Figure 11: Simulated scarcity effects for $K_2 = 3$, K_1 varying

(a) Firm 1 own-product $\omega_{1,t}^1(K_1, 3)$

(b) Firm 2 own-product $\omega_{2,t}^2(K_1, 3)$

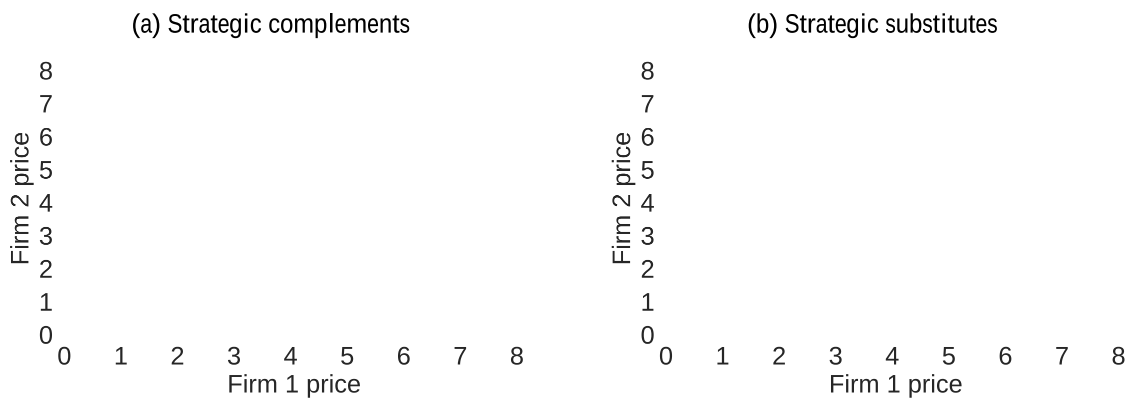


(c) Firm 1 competitor $\omega_{2,t}^1(K_1, 3)$

(d) Firm 2 competitor $\omega_{1,t}^2(K_1, 3)$

Notes: The simulations assume $= (1, 1)$, $_t$ 1 and logit demand with scaling factor $= 0.05$.

Figure 12: Strategic complements and substitutes in the stage game



Notes: The simulations assume $= (1, 1)$, $_t$ 1 and logit demand with scaling factor $= 0.05$, as well as $!^1_1 = !^2_2 = 4$. Panel (a) shows both firms' best response functions for $!^1_2 = !^2_1 = 4$. Panel (b) shows both firms' best response functions for $!^1_2 = !^2_1 =$ 4.

Figure 13: Simulated prices and scarcity effects

K = (5, 4)

(a) Price paths over time | (b) Own ! over time | (c) Competitor ! over time

K = (4, 4)

(d) Price paths over time | (e) Own ! over time | (f) Competitor ! over time

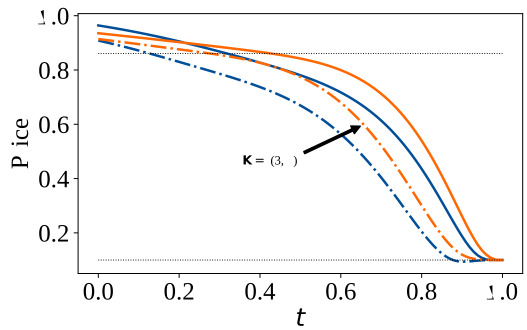
K = (3, 4)

(g) Price paths over time | (h) Own ! over time | (i) Competitor ! over time

Notes: The simulations assume = (1, 1), $_t$ 1 and logit demand with scaling factor = 0.05.

Figure 14: Price paths for varying levels of capacity

(a) Sale of a product with minimum inventory

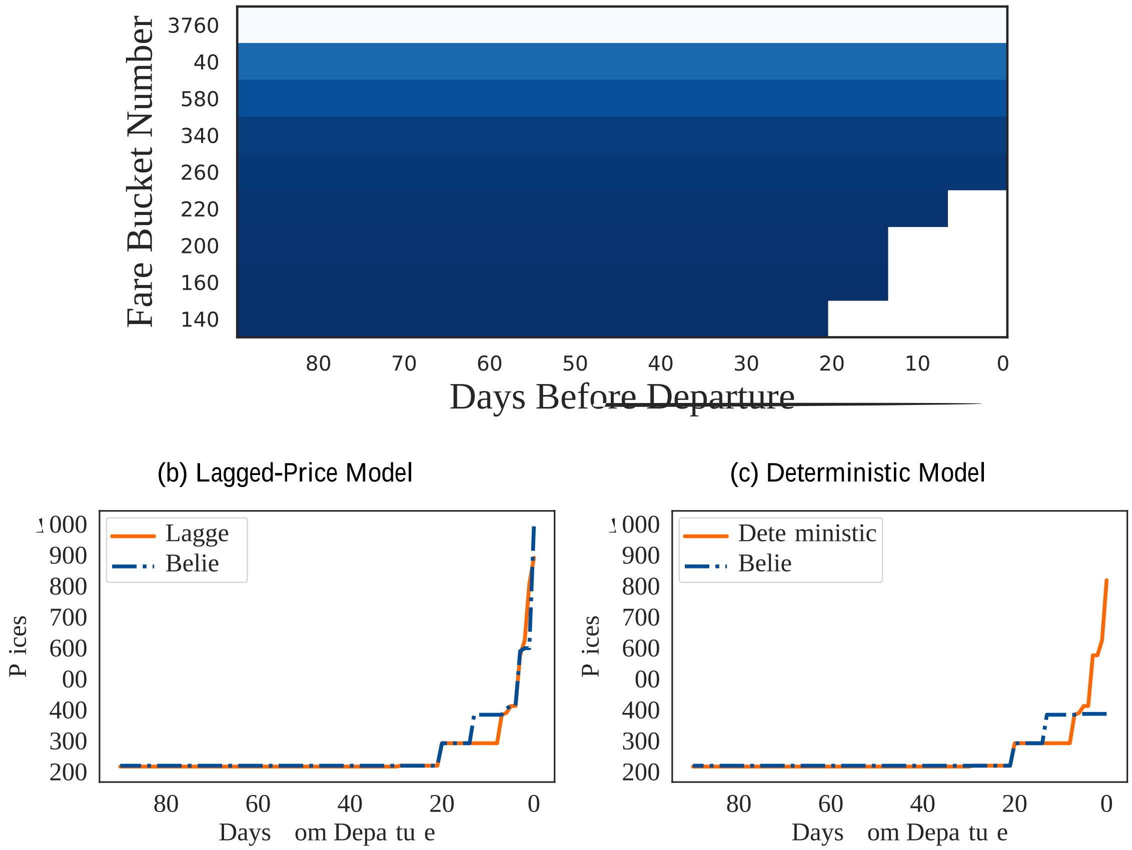


(b) Sale of a product without minimum inventory

Notes: These simulations correspond to logit demand with parameter values $_j = 1$, $= 1$, $= 10$ and scale factor $= 0.05$. Panel (a) shows both firm's price paths for $K = (3,5)$ and $K = (2,5)$. Panel (b) shows both firm's price paths for $K = (3,5)$ and $K = (3,4)$.

Figure 15: Heuristic Models Pricing Example

(a) Deterministic Model



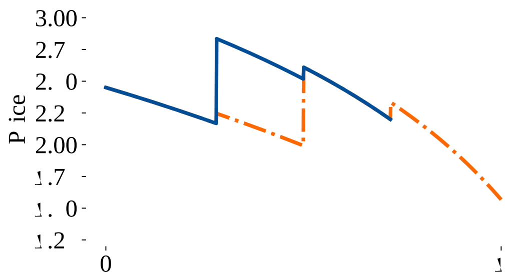
(b) Lagged-Price Model

(c) Deterministic Model

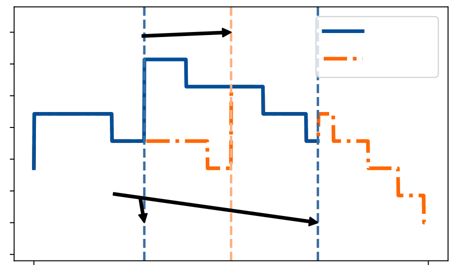
Note: Panel (a) shows a firm's fares and belief of the other firm's price over time for an instance of the simulation in the lagged-price model. Panel (b) shows a firm's fares and belief of the other firm's price over time for an instance of the simulation in the deterministic model.

Figure 16: Price Path Realizations comparing Benchmark model to Heuristics

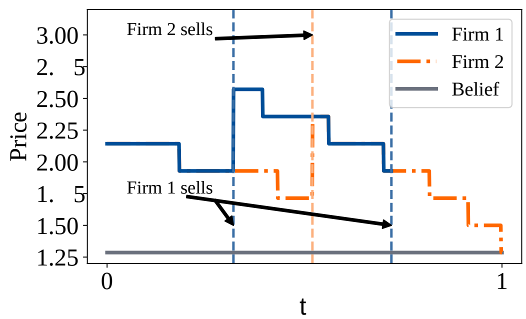
(a) Price paths for the benchmark model



(b) Price paths for the lagged algorithm



(c) Price paths for the deterministic algorithm



Notes: We assume demand follows a logit specification with an initial capacity vector of $K_0 = (2,2)$. Time is continuous for $t \in [0,1]$. There are three panels: panel (a) depicts the equilibrium price path for the benchmark model, panel (b) considers prices if firms use the lagged model, and panel (c) considers prices if firms use the deterministic model. The vertical lines mark realized sales times; the color denotes the firm that received the sale. These simulations correspond to the parameter values $_j = 1$, $= 1$, $= 1$, $= 10$ and $K_0 = [2,2]$. In the heuristic model, firms assume that the competitor prices at the level given by the grey line.

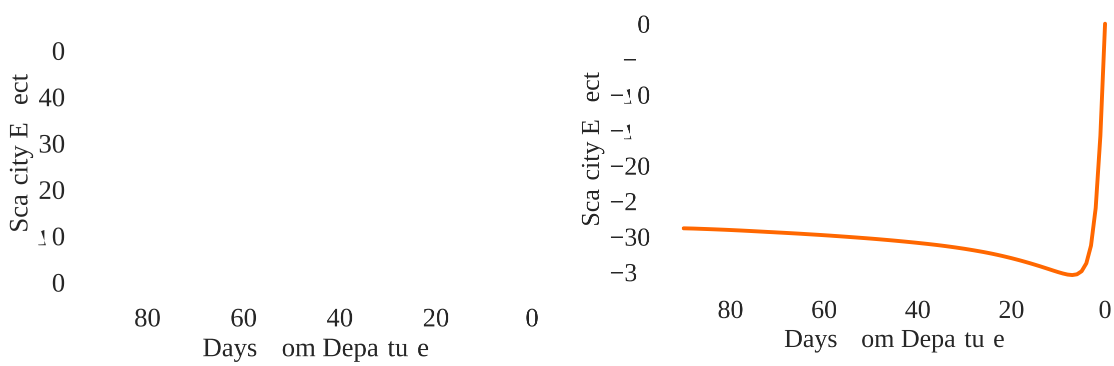
Table 4: Recreation of Table 3 with restricted initial capacity

	Price	Firm 1 Rev.	Firm 2 Rev.	CS	Welfare	Q	LF	Sellouts
Benchmark	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Deterministic	94.1	95.5	95.9	108.4	103.0	105.1	102.0	178.3
Lagged	102.0	100.3	101.2	104.4	102.9	100.6	100.2	104.0
Uniform	97.5	78.1	77.3	113.7	98.5	101.1	99.9	242.0

Note:

Figure 17: Recreation of Fig. 7 with restricted initial capacity

(a) Own Omega (b) Competitor Omega



Note: Panel (a) reports the own-firm scarcity effect over time for both firms. Panel (b) reports the cross-firm competitor scarcity effect over time for both firms.

Figure 18: Recreation of Fig. 8 with restricted initial capacity

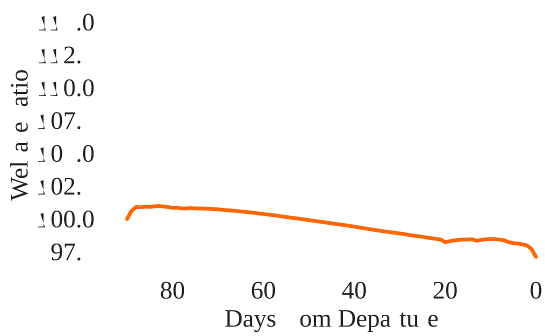
(a) Prices over Time

80 60 40 20 0

(b) Load Factors over Time

(c) Sellouts over Time

(d) Cumulative Welfare Comparison



Note: Panel (a) shows the average price over time for the benchmark, deterministic and uniform models. Panel (b) shows the average load factors over time for the same three models. Panel (c) shows the average sellouts over time for the same three models. Panel (d) shows the ratio of average cumulative welfare for the benchmark model with respect to the deterministic one, and for the benchmark model with respect to the uniform one.

Figure 19: Recreation of Fig. 9 with restricted initial capacity

(a) Cumulative CS Difference

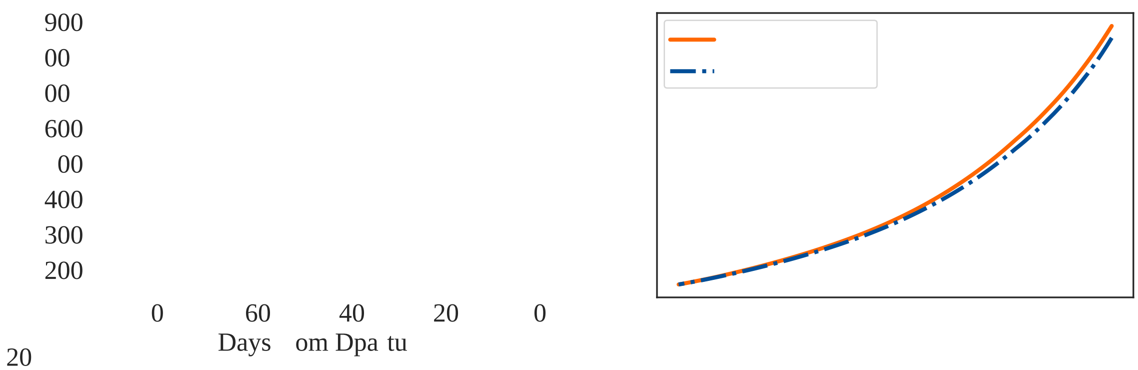
(b) Cumulative Revenue Difference

Note: Panel (a) reports the own-firm scarcity effect over time for both firms. Panel (b) reports the cross-firm competitor scarcity effect over time for both firms.

Figure 20: Recreation of Fig. 10 with restricted initial capacity

(a) Prices over Time

(b) Load Factors over Time

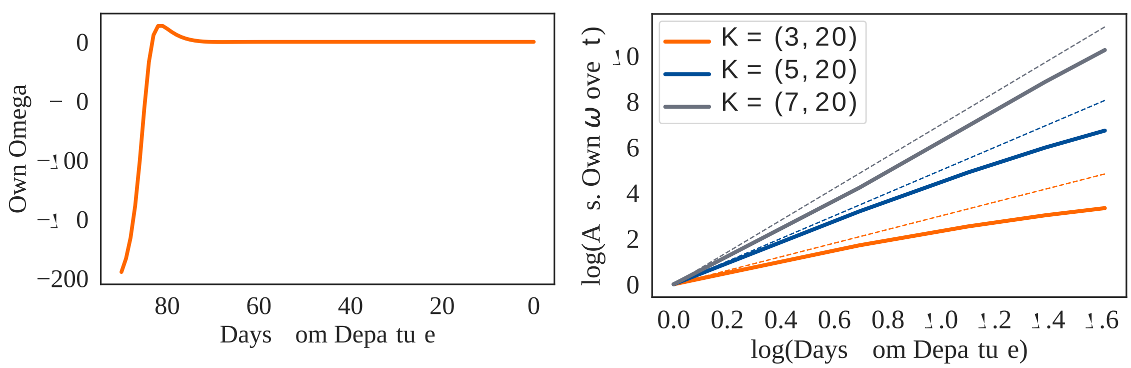


Note: Panel (a) shows the average prices over time for the two heuristic models. Panel (b) shows the average load factors over time for the same two models.

Figure 21: Example of a negative own Opportunity Costs

(a) Example own Omega over Time

(b) Log. Abs. Own ! over Time



Note: Panel (a) shows the own ! over time for a given state in one of our Benchmark solutions. Panel (b) shows the log of the absolute value of the own ! over time for three states in one of our Benchmark solutions. The dotted lines represent the behavior these curves would follow if the omegas were proportional to jT t j$^{min(K)}$.